APPLICATION OF COMPUTATIONAL FLUID DYNAMICS (CFD) TO FLOW
SIMULATION AND EROSION PREDICTION IN SINGLE-PHASE AND
MULTIPHASE FLOW

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There are two primary goals of this study: to determine the relative erosion resistance between plug tees and elbows for single-phase flow and to estimate the erosion in elbows for multiphase flow using a relatively simplistic approach.

For single-phase flow, a CFD-based erosion prediction procedure was applied to predict the erosion in elbows and plug tees for a broad range of flow conditions. This CFD-based erosion prediction procedure was significantly improved by implementing a stochastic particle rebound model. The penetration ratio between the plug tee (both End Region and Side & Corner Region) and the elbow was determined numerically by applying the implemented stochastic particle rebound model. Simulations illustrate that plug tees experience greater erosion than elbows for conditions where the carrier fluid has high density and viscosity, like water. The predictions agree with Bourgoyne’s (1989) experimental observations. Experimental erosion tests were conducted to validate simulation results obtained in air. Based on experimental data and simulation results, a simplified model was developed to calculate the penetration ratio between the Side &
Corner Region of the plug tee and the elbow. The value of 0.5 was recommended for the penetration ratio between the End Region of the plug tee and the elbow.

Immaturity of multiphase flow modeling and particle tracking determines that the single-phase CFD-based erosion prediction procedure can not be simply adopted for erosion prediction in multiphase flow. Consequently, a mechanistic and CFD combined approach was proposed to estimate the erosion in elbows for multiphase flow. The flow regimes covered by this study include bubbly flow, slug flow, and annular/annular-mist flow. The key of this approach is to identify a representative single-phase flow and to determine the effective sand mass ratio that result in an accurate erosion prediction for individual flow patterns. The single-phase CFD based erosion prediction procedure was employed to calculate the erosion for the resulting representative single-phase flow. The proposed erosion estimate approaches were validated by the experimental data of Bourgoyne (1989) and Salama (1998). Reasonable agreement was achieved between the predictions and the data.
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DEDICATION

I dedicate this work to my family: my grandfather, Lianxiang Chen, and father, Hechang Chen, whose encouragement and expectations inspired me to achieve this academic accomplishment that they will never be able to witness; my mother, Youpeng Ye, who taught me to be responsible and be dedicated to things that really matter; my wife, Qian Li, whose love is so important in my life and whose support is vital to my graduate studies; my parents-in-law, Ruren Li and Shu Zhou, and my siblings, Lijuan Chen, Xiangyun Chen and Xiangfu Chen. I also dedicate this work to my M.S. thesis advisor, Professor Zhili Zhao.

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CHAPTER I

INTRODUCTION AND BACKGROUND

Introduction

Many engineering industries have the need to transport fluids with entrained solid particles. In the oil and gas industry, the raw produced fluids are transported to refining stations from the reservoirs via piping systems and other equipment. In most cases, especially gas wells, the produced fluids contain sand and other solid particles. Erosion of material surfaces results from the impact of sand particles carried in the fluid, which can damage many piping devices and limit the reliable operation of piping systems.

Entrained solids can damage the inside surfaces of pipes, valves, fittings, and other components. The erosion damage of inside surfaces of piping devices caused by entrained solid particles is both extremely dangerous and expensive. Sand erosion may even result in the failure of the device and the piping system. Thus, the sand erosion damage requires the frequent replacement or repair of the device or component that is exposed and susceptible to the erosive environment. In order to avoid system shut downs and to keep the losses to a minimum, it is essential to study the erosion caused by solid particles. One effective way of controlling erosion is to accurately maintain operating conditions such as flow rates or velocities at acceptable levels. To keep the system operating safely, the relationship between flow conditions and erosion damage of the devices needs to be determined.
Complex geometries, such as elbows, plug tees, valves, pumps, chokes, and other fittings are commonly found in piping systems and equipment used by the petroleum industry. The common feature of the geometries is that there is a geometry transition, which changes the flow streamlines direction as well as the path of sand particles in the geometry. The particles entrained in the carrier fluids obtain momentum from the carrier fluid. When the carrier fluid and sand particles pass through these geometries, some of the particles cannot follow the streamlines. They cross streamlines and impinge the walls, which causes erosion to occur. Common geometries that are susceptible to erosion damage include elbows (shown in Figure I-1), expansions, and others. The consequences are expensive failures of the geometry in service. Prediction of erosion rate in the geometry allows estimating the service life of the geometry and predicting the possible failure of the geometry and the piping system.

Many pipe fittings and devices are specifically used in the transition section of the piping system to redirect the fluid. Examples include elbows and plug tees. The flow streamlines as well as fluid velocity profiles are changed drastically from straight pipe flow behavior. A large amount of sand particles can be driven toward the wall when the flow changes direction. Thus, elbows and plug tees may experience severe erosion damage under certain conditions. Figure I-1 also demonstrates the failure in an elbow due to sand erosion damage.

Two mechanisms contribute to the erosion resulting from particle impingements: direct impingement and random impingement. When the flow is redirected by a bounded geometry such as an elbow or plug tee, sand particles can acquire sufficient momentum from bulk fluid velocity components of the carrier fluid to cross the fluid streamlines,
which causes direct impingements on the wall. Random impingements occur when the particles are driven toward the wall by the turbulent fluctuations. The erosion in a straight pipe section mainly results from random impingements. Figure I-2 exhibits these two different impingement mechanisms.

![Figure I-1. Erosion Damage in an Elbow.](image)

![Figure I-2. Particle Direct and Random Impingement Mechanisms.](image)
Background

The sand erosion phenomenon is highly complicated. A wide range of parameters affect the erosion severity such as production flow rate, sand rate, fluid properties, sand properties, sand shape and size distribution, wall material of equipment, and geometry of the equipment. Among these factors, the momentum transfer process between carrier fluid and sand particles dominates the erosion process. A thorough understanding of the flow field and the interaction between the carrier fluid and solid particles is fundamental to perform the erosion investigation.

Since the flow field is affected by the geometry of the equipment, the sand erosion is also geometry dependent. Elbows are the most common geometry used to redirect fluids in piping systems. However, elbows are vulnerable to erosion. Field experience and studies (Wang, 2003) both show that long radius elbows are desirable to reduce erosion with respect to standard elbows in erosive environments. Commonly, plug tees are adopted in practice to reduce erosion especially if it is the case that space consideration is important and application of long radius elbows is not practical. Thus, industries have the need to determine the relative erosion severity of elbows to plug tees under a variety of flow conditions. Schematics of the standard elbow \( \frac{r}{D} = 1.5 \), where \( r \) and \( D \) are the turning curve radius and the pipe diameter of the elbow, respectively), long radius elbow, and plug tee are demonstrated in Figure I-3. One can easily deduce that in an erosive environment the erosion in plug tees is also a concern due to the sudden change of flow direction changing within the geometry. Because of the geometric complexity of plug tees as compared to elbows, it is expected that the erosion
phenomenon in plug tees is even more complicated than in elbows. Accordingly, the erosion phenomenon in plug tees is far less understood than elbows.

![Diagram of flow through standard elbow, long radius elbow, and plug tee](image)

(a) Standard Elbow  (b) Long Radius Elbow  (c) Plug Tee

**Figure I-3. Schematic of the Standard Elbow, Long Radius Elbow, and Plug Tee Geometries.**

Some models have been developed by the Erosion/Corrosion Center (E/CRC) and by others to predict erosion. Representative work has been done by McLaury (1996) and Salama (1998). Furthermore, the E/CRC has developed a computational fluid dynamics approach for generating erosion predictions. The procedure utilizes a commercially available computational fluid dynamics (CFD) code, CFX. This CFD-based erosion prediction procedure involves flow modeling, particle tracking, and erosion models and has been implemented into CFX by Edwards (2000). This comprehensive approach of erosion prediction can be used to study the effects of various parameters such as velocity conditions and geometry, as well as fluid and particle properties, on the erosion patterns associated with flow through various oilfield geometries for single-phase flow.
Multiphase flows, as well as single-phase flows, are commonly encountered in chemical as well as oil and gas industries. A variety of flow phenomena and flow regimes can be observed in multiphase flow, such as bubbly flow, annular flow, slug flow, and churn flow. Multiphase flow is characterized by the existence of the interface between different phases. The interface between phases can be described as irregular, transient, and stochastic. The interaction between phases can be extremely complicated. The mechanisms of some multiphase flow phenomena remain unknown to engineers and scientists. Industry has a strong desire to estimate and control erosion in different oilfield geometries under multiphase flow conditions to minimize the loss resulting from the sand erosion. Due to the difficulty in analyzing multiphase flow phenomena and sand particle behavior in multiphase flow, predicting sand erosion in multiphase flow conditions is much more challenging than in single-phase flow. Consequently, the capability of predicting erosion in multiphase flow is very limited.

**Research Goals**

The elbow geometry is broadly used when a flow direction transition is required. Due to the characteristics of the geometry, an elbow experiences severe erosion damage under certain flow conditions. For some cases, a plug tee is used instead of an elbow in order to reduce the erosion. However, it is unclear whether a plug tee leads to erosion reduction. Therefore, one of the primary goals of this work is to develop a mechanistic erosion prediction model that is used to calculate the relative erosion severity between plug tees and elbows for single-phase flow. The mechanistic erosion model can be used
as a guideline for industries to determine the appropriate pipe fitting geometry (elbow or plug tee) in an erosive environment.

In order to accomplish this task, several intermediate goals must be achieved. The sand erosion in elbows and plug tees must be obtained in a variety of single-phase flow conditions both analytically and experimentally. The flow conditions should be broad enough to cover normal operating conditions in industry practice. The effect of different parameters such as fluid properties and fluid flow rates, sand properties and sand production rates, geometry, and properties of pipe material must be investigated. The relative erosion severity ratio between plug tees and elbows can be determined from the erosion predictions. This relative erosion ratio finally leads to the development of the mechanistic erosion prediction models for plug tees.

Another objective of this work is to investigate sand erosion in oilfield geometries such as elbow in different multiphase flow regimes by applying the CFD-based erosion prediction model. This effort should be able to demonstrate the applicability of the current CFD-based erosion model in multiphase erosion prediction.

In order to achieve this goal, the following intermediate tasks have to be accomplished. The applicability of the CFD code to predict erosion in multiphase flow needs to be investigated in different multiphase flow regimes. For some flow conditions, a thorough simulation of the flow field is not achievable due to the extreme complexity of the multiphase flow phenomena and the limitation of current models. Therefore, appropriate alternative methods must be found in order to approximate the flow in different multiphase flow regimes and thus to approximately calculate the corresponding sand erosion.
Research Approach

Modeling and experimentation approaches are adopted to develop the mechanistic erosion prediction model for plug tees in single-phase flow. The core of the work resides in the modeling effort. The Computational Fluid Dynamics (CFD) based erosion prediction procedure developed by E/CRC is employed to predict the erosion in elbows and plug tees for a variety of flow conditions in single-phase flow. The CFD code utilized is CFX which is commercially available. This comprehensive erosion prediction procedure consists of three main steps: flow modeling, particle tracking and erosion prediction.

A CFD code with particle tracking capability, CFX-4, is applied in the first step to obtain the three-dimensional flow simulation within the geometry. The predicted flow field is used as the input information in the particle-tracking modeling to determine particle trajectories. A large number of particles (in the order of 10,000) are normally required in order to obtain a statistically representative erosion distribution on the geometry surface. Wall impingement information such as location, speed, and impact angle can be acquired from the particle trajectories. Finally, empirical erosion equations use the particle impingement information to calculate the local penetration rate on the geometry surface. The penetration rate is defined as pipe wall thickness loss rate due to the particle impingements. The experimentation in elbows and plug tees is primarily used to verify the erosion results obtained from the modeling efforts. The analysis of simulation results as well as experimental data finally leads to the development of the simplified erosion prediction model for plug tee for single-phase flows.
A modeling approach is applied to predict the sand erosion in multiphase flow. The CFD-based erosion procedure is utilized to simulate the multiphase flow field and erosion. First of all, efforts are made to investigate CFX’s capability of simulating multiphase flows in different flow regimes such as bubbly flow, slug flow, and annular flow. Due to the immaturity of the multiphase flow models to date, CFX is not able to provide accurate flow simulations for certain flow regimes. Simplifications must be made according to the flow characteristics of the multiphase regime of interest in order to make erosion prediction possible for these flow conditions. Approximate erosion prediction approaches for multiphase flow are developed based on the simplifications. Experimental data available in the literature is used to compare with predicted erosion, by which the proposed erosion approximation approaches are validated.
CHAPTER II
LITERATURE REVIEW

Introduction

The present research is directed at gaining a better understanding of the erosion in oilfield geometries such as elbows and plug tees in both single-phase and multiphase flows. This work focuses on the development of a simplified erosion prediction model to calculate the relative erosion severity between plug tees and elbows in single-phase flows. The amount of erosion depends on a multitude of factors. In order to provide an erosion model that is of practical use for production situations in the oil and gas industry, this simplified erosion model must be able to reflect the effects of the major flow parameters such as properties of fluids, particle properties and pipe size.

The current research is also dedicated to investigating the erosion in elbows for multiphase flows. Multiphase flow can be classified as bubbly flow, slug flow, churn flow, and annular flow according the characteristics of the interface between the phases. Each flow pattern is characterized by its unique interface structure as well as the mass and momentum transfer process between phases via the interface. Thus, flow regime is another major parameter that affects the erosion process as well as the quantity of erosion in multiphase flow.

In order to determine the erosion in different geometries for a wide range of flow conditions, two approaches can be taken: experimentation and modeling. A purely
The experimental approach is impractical since it requires the actual oilfield geometries and the corresponding scale flow loops for testing, and it takes a significant time to obtain measurable erosion only for one test in the dilute system. Thus, combined efforts of experimentation and modeling are taken to investigate the erosion in elbows and plug tees in single-phase flow to develop the simplified erosion model for plug tees. Major task is the modeling effort. Experimental data from testing on the test cells of small-scale elbow and plug tee geometries are used as a means to validate the simulation results from the modeling effort. For multiphase flow, only a modeling effort is considered in the present research to simulate the multiphase flow field and to predict the erosion. The modeling results are verified by data available in E/CRC and in the literature.

The purpose of this chapter is to review the literature that is involved in modeling erosion. The CFD-based erosion prediction procedure is adopted to predict erosion patterns in elbows and plug tees. This procedure consists of three major sections: flow modeling, particle tracking, and erosion calculations. So the literature review discusses erosion prediction methods, flow modeling, particle tracking, and erosion calculation models.

**Erosion Prediction Methods**

In order to combat the erosion problems caused by solid particles, a variety of erosion prediction methods have been developed. Most methods are based on a limited amount of experimental data, so these models are only applicable to specific conditions. Due to the lack of an accurate and general erosion prediction model, “rule of thumb” design guidelines are adopted for many situations in industrial practice, which usually
lead to overly conservative designs. The most well-known guideline in oil and gas industry is the American Petroleum Institute Recommended Practice 14E (API RP 14E) (1991). This guideline suggests a limiting flow velocity for erosive service. The guideline states that severe erosion should not occur if production velocities are maintained below this limit. The erosional velocity recommended by API RP 14E is given by Equation II-1:

\[ V_e = \frac{C}{\sqrt{\rho}} \]  

(II-1)

where \( V_e \) is the erosional velocity limit in ft/s, \( \rho \) is the carrier fluid density in lb/ft\(^3\), and \( C \) is a constant. The API guideline recommends that the value of \( C \) should be 100 for continuous service and 125 for intermittent service. In fact, the only variable accounted for directly in Equation II-1 is fluid density. API RP 14E is incapable of incorporating all the important factors involved such as fluid viscosity, properties of sand particles, and the sand production rate. Industrial applications have demonstrated that these parameters can have significant impacts on the erosion. API RP 14E has received much criticism for being extremely conservative. Svedeman and Arnold (1993) and Salama (1998) have introduced significant modifications to API RP 14E. McLaury and Shirazi (2000) have provided alternative methods, which include the effect of parameters such as properties of carrier fluid, sand particles, as well as material of pipe.

Significant efforts have been taken by researchers and engineers to study the erosion problem in single-phase flows and many erosion prediction models have been developed. For example, Nesic and Postlethwaite (1991) studied the local erosion in chokes by determining the local fluid velocity and particle impingement information. McLaury (1993) and Shirazi et al. (1995) developed erosion mechanistic models for
elbows. In another study, McLaury (1996) proposed a generalized erosion prediction procedure that involves flow simulation, particle tracking, and erosion prediction and studied the erosion in chokes. Wang et al. (1996) applied this approach to investigate the effects of elbow curvature on erosion rate in a three-dimensional geometry. The erosion in oilfield control valves was calculated by Forder (1998) using the flow simulation and particle tracking method. More recently, Edwards (2000) implemented the generalized erosion prediction procedure in a commercially available Computational Fluid Dynamics (CFD) code, CFX. This CFD-based erosion technique is a comprehensive procedure that is able to predict the erosion for a wide range of three-dimensional geometries. By applying this procedure, the maximum erosion for a certain case can be calculated, as well as the erosion profile on the surface of the geometry. In another study, Keating and Nesic (2001) applied the CFD approach to investigate the erosion-corrosion problems in U-bends.

Comparatively, the erosion prediction model development and prediction capability are very limited in multiphase flow. In the literature, there are few erosion models for multiphase flow, including Salama’s (1998) empirical model for elbows, Jordan’s (1998) semi-mechanistic erosion model, and the semi-mechanistic model for elbows by McLaury and Shirazi (1999). These models share a common feature that they were developed from and validated by limited available experimental data. Because of the variety of factors that are involved in the erosion problem in multiphase flow, it is not unusual that even the experimental data seems to be chaotic, so it is difficult to identify the primary parameters during the analysis of experimental data. Because the flow behavior and erosion are strongly controlled by the multiphase flow regime, Mazumder et
al. (2004) attempted to incorporate the flow pattern effect in a mechanistic erosion model for elbows.

The complexity of the combination of particle tracking and multiphase flow and the limitations of current CFD multiphase models determine that it is extremely challenging to study the erosion in multiphase flow numerically as thoroughly as single-phase flow. Consequently, there is no successful application of CFD in multiphase erosion prediction in literature to date.

**Flow Modeling**

**Turbulent Flow (Single-Phase) Modeling**

Most of the practical hydrodynamic fluid flows of interest are turbulent. Turbulent flow remains challenging to researchers to some degree although attempts to understand the physics of such flows, as well as efforts to predict and control them, have been ongoing for over 100 years. Fluctuations caused by the random motion of turbulent eddies introduce additional shear stresses (termed Reynolds stresses). The presence of Reynolds stresses causes significant difficulty in the prediction of turbulent flow behavior. Unlike laminar flow, there is no practical theory available to explicitly calculate flow parameters (mean velocity and fluctuation components, and Reynolds stresses). Approximations must be made to model the Reynolds stresses in order to obtain solutions to the Reynolds-Averaged Navier-Stokes (RANS) equations.

Attempts to model turbulent flow began with Prandtl (1925) ever since have been one of the most active topics of fluid mechanics research. White (1974) has summarized six different classes of such models, including zero-, one, and two-equation turbulence.
models, Reynolds stress models, large-eddy simulation method (LES), and direct numerical simulation (DNS).

The simplest turbulence models are zero-, one-, and two-equation models, which are categorized according to the required number of additional partial differential equations (PDEs) that must be solved in order to close the governing equation system. The common assumptions of these three classes of models are that turbulence or eddy viscosity exists and the Reynolds stress is expressed as a gradient diffusion term, similar to the shear contributions of time-averaged velocity components. In 1877 Boussinesq introduced the concept of an eddy viscosity. Based on the concept of eddy viscosity, zero-equation and one-equation models were postulated by Prandtl (1925, and 1945). Both in the zero-equation and one-equation models, a length scale must be used and is problem dependent. To overcome this shortcoming, researchers introduced two-equation models, such as the $k-\varepsilon$ models (where $k$ and $\varepsilon$ are the turbulence kinetic energy and its dissipation, respectively) and the $k-\omega$ (where $\omega = \varepsilon/k$ is the turbulence frequency) model. The two-equation standard turbulent kinetic energy-turbulent dissipation rate ($k-\varepsilon$) model of Jones and Launder (1973) is by far the most used turbulence model. Yakhot and Orszag (1986) presented a modification to the standard $k-\varepsilon$ model that uses renormalization group (RNG) theory to analyze velocity fluctuations. The RNG turbulence model developed by Yakhot and Orszag (1986) is a refinement of the two-equation $k-\varepsilon$ model of Jones and Launder.

The Reynolds stresses are solved directly in the Reynolds stress turbulence models. Each of the Reynolds stresses in the governing equation requires modeling efforts. As compared to two-equation models, the Reynolds stress models are more
complex and computationally expensive but have the potential of greater accuracy and wider applicability. The model devised by Launder et al. (1975) is the most well known and most thoroughly tested second-order closure model.

If previously discussed turbulence models can be classified as “classical” models, then LES and DNS belong to “modern” methods. LES is a turbulence model where the mean flow as well as large eddies are solved from the time-dependent flow equations and where the effects of smaller scale eddies are modeled (Voke and Peter, 1983). The scale of eddies to be solved is determined by a scale filter which needs to be provided in the LES. LES is presently at the research stage and the calculations are too costly for general purpose computation at present. Anticipated improvements in computer hardware make LES practical for engineering applications in the future.

Rather than LES, with DNS methods the motion of turbulent eddies is computed throughout the entire flow domain without resorting to modeling approximations. From this viewpoint, LES is somewhat midway between RANS and DNS. Although DNS is extremely computationally expensive, it has strong potential to supply values of turbulence properties that cannot be measured in the laboratory owing to the absence of suitable experimental techniques. Hence DNS can be applied to validate the empirical coefficients of classical turbulence models and be used to guide the development of more accurate classical models.

**Multiphase Flow Modeling**

Multiphase flows are encountered in many processes in industrial operations. Numerous examples can be found in the chemical, petroleum, and power-generation industries. In multiphase flow the flow-constituents or phases are mixed at larger than
molecular length scales and may have velocities, pressures, and temperatures relative to each other. Due to the inherent complexity of multiphase flows, from a physical as well as numerical point of view, “general” applicable computational fluid dynamics (CFD) codes are non-existent. The reasons for the lack of fundamental knowledge on multiphase flows are three-fold (van Wachem and Almstedt, 2003):

(1) Multiphase flow is a very complex physical phenomenon where many flow types can occur (gas-solid, gas-liquid, liquid-liquid, etc.) and within a certain flow type several flow regimes are able to exist (bubbly flow, annular/stratified flow, slug flow, etc.);

(2) The complex physical laws and mathematical treatment of phenomena occurring in the presence of the two phases (interface dynamics, coalescence, break-up, drag, etc.) are still largely underdeveloped. For example, to date there is still no agreement on the governing equations. In addition, proposed constitutive models are empirical but often lack experimental validation for the conditions they are applied under;

(3) The numerics for solving the governing equations and closure laws of multiphase flows are extremely complex. Very often multiphase flows show inherent oscillatory behavior, requiring costly transient solution algorithms. Almost all CFD codes apply extensions of single-phase solution procedures, leading to diffusive or unstable solutions, and require very short time-steps, or Courant, Freidricks, Levy (CFL) numbers.

In spite of the major difficulties mentioned above, significant progress has been made in multiphase flow CFD. Various multiphase flow CFD models that are applicable
for certain ranges of applications have been developed. These models can be divided in
two categories: Eulerian-Lagrangian model and Eulerian-Eulerian model. In general, the
Eulerian-Lagrangian approach is suitable for continuous-dispersed flows. By extending
Single-Phase Navier-Stokes Equations to multiphase flow, Ishii (1975) pioneered a
general approach loyal to conservation laws for each phase and derived multiphase fluid-
fluid governing equations on which the Eulerian-Eulerian approach is based. Several
Eulerian-Eulerian sub-models including homogenous model, drift flux model and multi-
fluid model are derived according to the number of governing equations that are solved in
the model. The homogenous model assumes that all phases are well mixed and therefore
travel at the same actual velocity. Thus only one set of momentum equations is solved
while the mass conservation equation is modeled for each phase. The Drift flux (or
Mixture) model is somewhat similar to the homogenous model except that velocity
slippage between phases is computed using an algebraic relationship and is based on the
assumption of local momentum equilibrium. Two-fluid (or Multi-fluid) model is a
“complete” multiphase flow model by which the mass as well as momentum equations
are solved for each phase by taking as many physical effects as possible into account
(Calay et al., 2003). In addition, the Inter-Phase Slip Algorithm (IPSA) method that
allows inter-phase momentum and mass transfer was firstly described by Spaling and
reported by Baghdadi (1979) in his PhD dissertation. IPSA is broadly applied when using
the drift flux model and the two-fluid model. More recently, Lahey (2000) developed a
multi-field model of two or more phases flows. This model assumes the individual fields
may represent either separate fluids and phases, or geometrically/structurally distinct
flow configurations within a given physical fluid or phase.
Although the Eulerian-Eulerian multi-fluid model is the most complete multiphase flow model, it demands a set of accurate constitutive closures that correlate inter-phase transport processes to provide meaningful simulations. Extensive and important discussions of closure correlations of the multi-fluid model are conducted by Drew and Lahey (1979), and Drew (1983). Unfortunately, constitutive closures are a problem, or at least multiphase flow regime dependent. On the other hand, the underlying assumption of the multi-fluid model is that phases are interpenetrating and averaged properties are solved at the local unit volume. Thus the “drag” term in the multi-fluid model is essentially suitable for continuous-dispersed flow. With much better understanding of bubble dynamics than other flow regimes (annular flow, slug flow, etc.), it is not surprising that most advancements of multiphase flow modeling have been related to bubbly flow. Yet, a doubt is thrown upon the multi-fluid model in simulating continuous-continuous flows where the interfacial shear instead of drag is dominant. More recent advanced development of CFD modeling for continuous-continuous flow includes: Anglart and Podowski (1999) derived the interfacial closures for slug flow that reflect the geometry of Taylor bubbles and its interfacial interactions; Antal (2000) proposed an annular flow modeling approach that takes the local liquid entrainment and deposition into account. These successful attempts were based on the application of Lahey’s (2000) multi-field multi-fluid model.

The subject of multiphase turbulent flow is not as well developed as single-phase turbulent flow. There is no industrial standard model, like the single-phase k – ε model, which is known to perform reasonably well to engineering accuracy for a wide range of applications. Ishii’s effort (1975) to formulate constitutive relations for the Reynolds
stresses in the two-fluid model marks the major breakthrough of multidimensional two-phase flow CFD modeling. One of the early works on turbulence modeling for this purpose was performed by Drew and Lahey (1982) who applied mixing length theory to analyze the phase distribution in vertical bubbly pipe flows. Lee et al. (1989) were the first to use CFD to predict phase distribution by applying the $k-\varepsilon$ model to bubbly flows. The constitutive relations necessary to adapt the $k-\varepsilon$ model to bubbly flows were investigated by Lopez de Bertodano et al. (1994). Recently, CFD modeling of multiphase turbulent flow has been one of most active area of fluid mechanics.

**Particle Tracking**

The motion of particles in viscous fluids is an old and complex problem. Initially, Stokes (1851) studied simple harmonic oscillations of a sphere and derived an equation for the forces exerted on the sphere by the fluid. By applying Newton’s second law, Stokes proposed a differential equation to describe the motion of the sphere in a fluid:

$$
\frac{\pi d_p^3 \rho_p}{6} \frac{dV_p}{dt} = 3\pi \left( \frac{d_p^3}{12} \rho_f \right) \frac{dV_p}{dt} + \frac{\pi d_p^3 \rho_f}{12} \frac{dV_p}{dt}
$$

$$
+ \frac{3}{2} \sqrt{\pi \mu} \rho_f d_p^3 \int_{t_0}^{t} \frac{1}{\sqrt{t - t_p}} \left( \frac{dV_f}{dt} - \frac{dV_p}{dt} \right) d\tau_p + F_e
$$

where $d_p$ is the particle diameter; $V_p$ is the particle velocity; $\rho_p$ and $\rho_f$ are the particle and fluid density, respectively; $\mu$ is the dynamic viscosity; and $\tau_p$ is the particle relaxation time. The left-hand side of Equation II-2 is the force required to accelerate the particle. The terms on the right-hand side of Equation II-2 accumulate the forces exerted on the particle. Term I is the drag force or viscous resistance according to Stokes’ law.
Term II accounts for the added mass effect. The added mass effect is due to the fluid around the particle that must be accelerated in addition to the particle itself. Term III is known as the history force which accounts for the deviation in the flow pattern from steady state. Landau and Lifshitz (1959) have performed the detailed derivation of the complex acceleration history term. Term IV is the sum of external or body forces such as gravitation or buoyancy. Stokes’ equation is a major breakthrough in the analysis of particle motion submerged in a fluid. But it should be noted that Equation II-2, in which the convective term is neglected, is only valid for relatively small particle Reynolds numbers.

Odar and Hamilton (1963) extended Stokes’ particle motion equation to higher particle Reynolds numbers. They introduced modifications such as empirical constants in the added mass and history terms to incorporate the convective effect of the fluid. Odar and Hamilton also included a coefficient expression in the drag term that is known as drag coefficient $C_D$.

The hidden assumption of Equation II-2 is steady state flow. Tchen (1947) has extended this equation to unsteady flows. Tchen also included a term to account for the fluid pressure gradient:

$$\frac{\pi d_p^3 \rho_p}{6} \frac{dV_p}{dt} = 3\pi\mu d_p (V_f - V_p) + \frac{\pi d_p^3 \rho_f}{6} \frac{dV_f}{dt} + \frac{\pi d_p^3 \rho_f}{12} \left( \frac{dV_f}{dt} - \frac{dV_p}{dt} \right) $$

$$ + \frac{3d_p^2 \sqrt{\pi \rho_f \mu}}{2} \int_0^t \frac{1}{\sqrt{t - \tau_p}} \left( \frac{dV_f}{dt} - \frac{dV_p}{dt} \right) d\tau_p + \sum F_e $$

(II-3)

The second term on the right side of Equation (II-3) is the added mass force term. Tchen made several assumptions in the formulation of Equation (II-3):
1) Particles are spherical in shape and relative velocity between the fluid and the particle is small so that Stokes’ drag is applicable;

2) Particles are small as compared to the smallest length scale of the so that shear flow effect is negligible;

3) Particles do not affect the flow;

4) Particles do not interact due to low particle concentration.

Saffman (1965) examined the turbulence shear effect on particle and introduced a lift force to the particle motion equation.

**Erosion Calculation Models**

Lagrangian particle tracking models not only determine the manner in which the particle interacts with the fluid but also indicate all the possible interactions between the particle and the target material. This information is closely related to the amount of erosion that may occur because the erosion primarily depends on the speed and angle of the impacting particle. Many correlations have been developed to quantify the erosion ratio that is defined as the mass loss of pipe wall due to erosion to the mass of sand. Finnie (1978), Benchiata (1980), and Sundarajan (1983) have presented a similar model, shown as Equation (II-4), to determine the erosion ratio using the particle impact speed:

\[
\text{ER} = A V^n
\]

where \(\text{ER}\) is the erosion ratio; \(A\) and \(n\) are empirical constants that are determined by the properties of sand particles as well as the target material; and \(V\) is the magnitude of the particle impact velocity. Different values of the velocity component \(n\) were obtained by Finnie (1960), Hutchings *et al.* (1976), and Tilly (1979), which implies that the erosion
ratio depends on factors other than the particle impact velocity. Tilly (1979) showed that
different particle impact angles resulted in various erosion ratios and that different
erosion ratios were detected for different materials with the same particle impact angle.
Tilly (1979) developed a model that incorporates the effects of particle impact angle as
well as material properties:
\[ ER = J \cos^2 \theta + K \sin^2 \theta \]  \hspace{1cm} (II-5)
where \( J \) and \( K \) are coefficients that depend on material properties and \( \theta \) is the particle
impact angle. According to Tilly, \( J \) is zero for completely brittle materials and \( K \) is zero
for completely ductile materials.

Finnie (1978) also presented an empirical erosion model that incorporated the
effects of particle impact speed and angle, as well as the pipe wall material yield stress:
\[ ER = \begin{cases} 
\frac{c \rho_w V^2}{2 \sigma_0} \left( \cos \theta - \frac{3}{2} \sin \theta \right) \sin \theta & \text{for } \theta \leq 18.5^\circ \\
\frac{c \rho_w (V \sin \theta)^2}{12 \sigma_0} \cot^2 \theta & \text{for } \theta > 18.5^\circ 
\end{cases} \]  \hspace{1cm} (II-6)
where \( c \) is an empirical constant, the value of which is nominally taken to be 0.5; \( V \) and
\( \theta \) are the particle impact speed and angle, respectively, \( \rho_w \) is the density of the target
material, and \( \sigma_0 \) is the target material plastic flow stress.

Bergevin modified Equation II-6 by introducing a critical particle impingement
speed, \( V_c \), below which erosion is assumed to be negligible:
\[ ER = \begin{cases} 
\frac{\rho_w (V \sin \theta - V_c)^2}{2 \sigma_0} \left[ V \cos \theta - \frac{3}{2} (V \sin \theta - V_c) \right] & \text{for } \theta \leq 18.5^\circ \\
\frac{\rho_w (V \sin \theta - V_c)^2}{12 \sigma_0} \cot^2 \theta & \text{for } \theta > 18.5^\circ 
\end{cases} \]  \hspace{1cm} (II-7)
Bergevin’s modification eliminates the empirical constant $c$ in Equation II-6. Some ambiguity still remains in determining the critical impingement speed.

The combination of the study by Hashish (1988) and Bitter (1963) suggests another erosion prediction approach that assumes a cutting component (important at small impact angles) and a deformation component (important at larger angles). Similar to the Finnie (1978) model, the Hashish/Bitter approach takes into account impact velocity and impact angle, as well as the wall material properties. In addition, more factors such as particle roundness, Possion ratio and Young’s modulus of the particle and wall material are considered.

Ahlert (1994) and McLaury (1996) presented erosion models for carbon steel and aluminum, respectively. The model equations relate impact speed and angle, material properties, and particle roundness to the magnitude of erosion ratio.
CHAPTER III
CFD-BASED EROSION PREDICTION PROCEDURE

The CFD-based erosion prediction procedure developed by Edwards (2000) is applied to predict the erosion rate in elbows and plugged tees for single-phase flow. This comprehensive procedure has three major steps: flow modeling, particle tracking, and erosion prediction. This chapter discusses this procedure in detail and each topic with relevant theories.

Computational Flow Modeling (Single-Phase)

The first step of the CFD-based erosion prediction procedure is to obtain the flow field simulation within the geometry. A commercially available CFD code, CFX-4, is applied to perform the flow simulation. Several turbulence flow models for single-phase flow are included in CFX-4, such as Standard, Low Reynolds Number and RNG versions of Turbulent Kinetic Energy-Dissipation (k–ε) models, Differential and Algebraic Reynolds Stress models, and a Reynolds Flux model. For details of the various turbulence models available, the reader is referred to the vast engineering literature on this topic.

The finite volume differential technique is adopted in CFX-4. The continuous flow domain is built using multi-block structured grids and is divided into discrete
control volumes. The flow governing equations (also termed as Navier-Stokes equations) are solved numerically on each node of the discretized flow domain.

The governing equations of flow employed in CFX-4 are discussed in this section. Since this research assumes that heat transfer process is negligible during the erosion process, only continuity and momentum equations are presented. The continuity and momentum equations are given in Equations (III-1) and (III-2) respectively for single-phase flow:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho U) = 0 \tag{III-1}
\]

\[
\frac{\partial (\rho U)}{\partial t} + \nabla \cdot (\rho U \otimes U) = \mathbf{B} + \nabla \cdot \left( - \rho \overline{u' \otimes u'} + \sigma \right) \tag{III-2}
\]

where \( \rho \) is the fluid density; \( U \) is the instantaneous velocity vector; \( \mathbf{B} \) is the body force; \( \overline{u' \otimes u'} \) is the Reynolds stress; and the stress tensor, \( \sigma \), is given by

\[
\sigma = -\frac{p}{\rho} \mathbf{I} + \frac{\mu}{\rho} \left[ \nabla U + (\nabla U)^T \right] \tag{III-3}
\]

where \( p \) is the local fluid pressure and \( \mathbf{I} \) is the identity matrix. The Reynolds stresses \( \overline{u' \otimes u'} \) is modeled by applying available turbulence flow models within CFX-4.

**Particle Tracking**

Upon obtaining the accurate flow field simulation, CFX-4 uses a “coupling” method to numerically predict trajectories of solid particles. During the “coupling” process, a Lagrangian approach is utilized to track trajectories of solid particles. There are two coupling methods: one-way coupling and two-way coupling. The one-way...
coupling method assumes that the flow field of the continuous carrier fluid is dominant and determines the motion of the dilute dispersed solid particles. In two-way coupling, the interaction between the carrier fluid and the particles is accounted for to calculate the continuous flow field as well as the trajectories of solid particles. For flow conditions of interest in oil and gas production, the sand concentration is fairly small so that the motion of sand particles has a negligible effect on the carrier fluid flow. Thus, one-way coupling method is employed to calculate sand particles trajectories in this research.

Within the particle transport model, a large amount of randomly distributed particles are usually tracked in order to represent the total flow of certain mass of the dispersed phase. The outputs of the particle tracking modeling are the complete trajectories of the tracked particles within the flow field. This section discusses the particle transport in the continuum carrier fluid.

**Position Equation**

CFX-4 uses the physical domain in the user interface and computational domain during the simulation including flow calculations as well as particle tracking. The user uses the physical domain to generate the physical flow domain, specify physical boundary conditions, and visualize simulation results. The CFX-4 solver uses the computational domain to perform the calculation of transport equations. The Jacobian transformation matrix bridges the physical domain and computational space (domain).

A particle is traced in the computational space in order to predict its trajectories. In the Lagrangian approach, a particle’s velocity is both space and time related. Equation III-4 describes the relation between the particle position in computational space and particle velocity:
where $x_p^{(c)}$ is the particle’s position vector in computational space, $t$ is time, and $V_p^{(c)}$ is the particle’s velocity vector in computational space.

The Jacobian of the coordinate transformation can be determined for each of the vertices of the control volumes. To ensure this transformation is continuous across control volume boundaries, the Jacobian is calculated continuously from the discrete information available directly on edges of control volume faces using Hermite interpolation within each block of the grid. The computational space velocity is obtained from the physical space velocity of the particle by calculating the Jacobian and taking the form of:

$$V_p^{(c)} = \left( \frac{\partial x_p^{(c)}}{\partial t} \right)^{-1} V_p$$

where $V_p$ is the particle physical space velocity vector.

**Momentum Transfer Equations**

According to Newton’s second law of motion, the particle physical space velocity vector is determined by Equation (III-6):

$$\pi d_p^3 \rho_p \frac{dV_p}{dt} = F_D + F_p + F_B + F_A + F_R$$

where left-hand side of Equation III-6 is the force required to accelerate the particle, $d_p$ is the particle diameter, $\rho_p$ is the sand density. The drag force that is exerted on the particle by the fluid is a dominant component among all the forces in the right-hand side
of Equation III-6. The drag force exists due to the presence of the local relative (slip) velocity between the particle and the fluid and is given by

\[ F_D = -\frac{\pi d_p^2 \rho_f C_D}{8} \left| V_p - V_f \right| (V_p - V_f) \]  

(III-7)

where \( V_f \) represents the fluid velocity at the current particle physical location and \( \rho_f \) is the local fluid density. The drag coefficient, \( C_D \), is given by

\[ C_D = \frac{24}{\text{Re}_s} \left( 1 + 0.15 \text{Re}_s^{0.687} \right) \]  

(III-8)

where \( \text{Re}_s \) is the particle slip Reynolds number and is defined by

\[ \text{Re}_s = \frac{\rho_f (V_p - V_f) d_p}{\mu} \]  

(III-9)

where \( \mu \) is the dynamic viscosity of the carrier fluid.

The right-hand side of Equation III-6 also includes large pressure gradient force \( (F_P) \), buoyancy force \( (F_B) \), added mass force \( (F_A) \), as well as a rotating coordinates term which accounts for both centrifugal and Coriolis effects \( (F_R) \). Equation III-10 through Equation III-12 give mathematical representations of \( F_P \), \( F_B \), and \( F_A \), respectively. Due to the stationary geometries considered in the current research, the rotating coordinates term is not discussed in this section.

**Pressure Gradient:** \[ F_P = -\frac{\pi d_p^2}{4} \nabla p \]  

(III-10)

**Buoyancy:** \[ F_B = \frac{\pi d_p^3}{6} (\rho_p - \rho_f) g \]  

(III-11)
**Added Mass:**

\[
F_A = -\frac{\pi d_p^3}{12} \rho_p \frac{dV_p}{dt}
\]  

(III-12)

In the pressure gradient term given by Equation III-10, \( \nabla p \) is the local pressure gradient in the carrier fluid. The pressure gradient force is typically small when the particles and the carrier fluid are of similar density. The buoyancy force in Equation III-11 is necessary to be included when there is a significant difference in the densities of sand particles and the carrier fluid and when the consideration of gravitational effects is desired. The added mass effect described by Equation III-12 is due to the fluid around the particle that must be accelerated in addition to the particle itself. When relative motion between the particles and the carrier fluid occurs, the fluid in the immediate vicinity of the particle tends to be accelerated with the sand particle. This results in resistive force acting on the particle and is termed as the “added mass” force.

**Turbulent Particle Dispersion**

In the turbulent flow field, forces due to the turbulence velocity components can have a strong influence on the solid particle trajectories. In CFX-4, the turbulent particle dispersion model accounts for the turbulence effect on the particle motion. In turbulent particle dispersion modeling, the approach taken assumes that the continuum flow field consists of succession of turbulent structures (eddies). Each individual eddy has its own unique length and time scales which depend on the local turbulence intensity. Particles interact with each of the eddies that are encountered when particles travel through the continuum fluid, which causes particles to deviate from the trajectory as predicted by Equation III-6.
Gosman and Ioannides (1981) approach is adopted by CFX-4 to simulate the effects of turbulent dispersion, which assumes that turbulent fluctuations of velocity components possess a Gaussian probability distribution defined by a mean of zero and a standard deviation of \( q \). By assuming isotropic turbulence, the value of \( q \) is determined by Equation III-13:

\[
q = \sqrt{u'^2} = \sqrt{v'^2} = \sqrt{w'^2}
\]  

(III-13)

where \( u' \), \( v' \), and \( w' \) are turbulence flow fluctuating velocity components. The equation for turbulent kinetic energy per unit mass is given by Equation III-14:

\[
k = \frac{1}{2} \left( u'^2 + v'^2 + w'^2 \right)
\]  

(III-14)

Thus, Equation III-13 can be rewritten in terms of turbulent kinetic energy as shown in Equation III-15:

\[
q = \sqrt{\frac{2}{3} k}
\]  

(III-15)

Following this procedure, the velocity of a given eddy is determined, and it is assumed that the eddy will maintain this velocity until its lifetime expires. Thus, random sampling of this distribution at appropriate points in the trajectory calculations yields the instantaneous velocity:

\[
U = u + u'
\]  

(III-16)

Length and lifetime scales of an eddy and the interaction time between the particle and the fluid are important considerations that must be made in particle dispersion modeling. The interaction time, \( T_i \), is dictated by either of the following possible events:
1) The relative velocity between the particle to the fluid is sufficiently small so that the particle remains inside the eddy during the entirety of the eddy lifetime $T_e$, or

2) The relative velocity between the particle and the fluid is sufficiently large so that the particle crosses the eddy before it expires in some crossing time $T_R$, which is less than $T_e$.

The interaction time is therefore the smaller value of either the eddy lifetime or the crossing time.

$$T_i = \min(T_e, T_R) \quad (III-17)$$

It is assumed that the length scale of the randomly sampled eddy, $L_e$, is equal to the one of the local turbulence dissipation, $L_T$, which is defined as a function of turbulent kinetic energy, $k$, and dissipation rate, $\varepsilon$.

$$L_e = L_T = C_{\mu}^{0.5} \frac{k^{1.5}}{\varepsilon} \quad (III-18)$$

where $C_{\mu}$ is a turbulence model constant. The eddy lifetime is thus determined by

$$T_e = \frac{L_e}{|u'|} \quad (III-19)$$

The particle crossing time is estimated by Equation III-20:

$$T_R = -\tau_p \ln \left[ 1 - \frac{L_e}{\tau_p |u_f - u_p|} \right] \quad (III-20)$$

where $\tau_p$ is the particle relaxation time and is defined as

$$\tau_p = \frac{4 \rho_p d_p}{3 \rho_f C_D |u_f - u_p|} \quad (III-21)$$
Equation III-21 has no solution in cases where $L_c > \tau_p |u_f - u_p|$ and it is therefore assumed that the particle is “trapped” by the eddy and thus $T_i = T_c$.

The previously discussed procedure is applied to all the sand particles that travel through the continuous flow field. If a large number of sand particles are tracked using this procedure, the overall effect of the turbulent dissipation on particle motion induced by the continuum fluid can be reasonably simulated by randomly selecting a large number of Gaussian distributed local turbulent flow velocity fluctuations.

**Near-Wall Particle Behavior (Squeeze Film)**

When a particle is approaching the wall, it must penetrate through the near wall flow region before the particle impinges the wall. Very near the wall there is a quasi-static layer of fluid. The particle must penetrate the fluid in this layer before it reaches the wall. This layer is termed by Clark and Burmeister (1992) as the “squeeze film”. Clark and Burmeister (1992) studied the effect of squeeze film on particle impingement characteristics and developed a model that simulates the influence of the squeeze film on the particle’s impact velocity.

According to the Clark and Burmeister (1992), the particle is able to penetrate the squeeze film only if the particle Reynolds number, $Re_p$, is larger than a critical particle Reynolds number, $Re_p^*$; otherwise the particle does not impinge the wall and the particle impingement is discarded. The particle Reynolds number is defined as

$$Re_p = \frac{\rho_p V_i d_p}{\mu} \quad (III-22)$$
where $V_i$ is the particle velocity component perpendicular to the wall before impact. The critical particle Reynolds number is given by

$$Re_p^* = \frac{12 \xi^2}{a}$$  \hspace{1cm} (III-23)

where $\xi$ is a dimensionless constant that depends on particle shape. The value of $\xi$ is equal to 10, as suggested by Clark and Burmeister.

$$a = 8 \left[ 2 \left( \frac{\rho_p}{\rho_f} \right) + f_{av} \right]$$  \hspace{1cm} (III-24)

where $f_{av}$ is a constant equal to 1.0, as suggested by Clark and Burmeister.

If the particle Reynolds number is large enough so as to penetrate the squeeze film, the squeeze film reduces the particle's perpendicular velocity component by a factor $\Gamma$, or

$$V_n = \Gamma V \sin \theta$$  \hspace{1cm} (III-25)

where $V_n$ is the normal velocity component after the squeeze film effects are considered and is used as the normal component of impingement velocity in erosion calculations. $\Gamma$ is given by

$$\Gamma = \frac{a}{a + \xi \left( 1 - \frac{Re_p^*}{Re_p} \right)}$$  \hspace{1cm} (III-26)

Similar to approaching the wall, the particle must penetrate through the squeeze film before it can leave the near wall region and rebound. The squeeze film model, which was implemented into the CFX particle tracking model (Edwards, 2000), is applied to the particle when it is approaching the wall before impact as well as leaving the wall after impact. Thus, the squeeze film calculations are performed twice for each impingement.
that occurs. The squeeze film may have significant influences on the particle’s impingement speed and angle especially in situations where the carrier fluid has high density and high viscosity like water, and the effects are negligible when the fluid is a low-density gas.

**Particle-Wall Interaction**

There is a momentum loss of the particle accompanied with the impingement on the wall. The level of particle momentum loss during the impingement can be accounted for by restitution coefficients that are the ratios of particle velocity components after impingement to the corresponding components before impingement.

In CFX, it is assumed that restitution effects apply only to the normal component of velocity by default. However, Forder *et al.* (1998), and Grant and Tabakoff (1975), demonstrated that the values of restitution coefficients of both perpendicular and parallel components depend on the particle impingement angle. Forder *et al.* provided the following correlations for perpendicular and parallel restitution coefficients based on impingement testing using AISI 4130 carbon steel:

\[
e_{\text{per}} = 1 - 0.4159 \theta + 0.5994 \theta^2 - 0.292 \theta^3 \quad \text{(III-27)}
\]

\[
e_{\text{par}} = 1 - 2.12 \theta + 3.077 \theta^2 - 1.1 \theta^3 \quad \text{(III-28)}
\]

where \(e_{\text{per}}\) and \(e_{\text{par}}\) are perpendicular and parallel coefficients of restitution, respectively. Grant and Tabakoff (1975) treated the rebound dynamics of the particles in a statistical sense. Based on experimental data (for 2024 Aluminum and 200 µm sand particles), Grant and Tabakoff postulated the mean values as well as the standard deviations of the
coefficients of restitution ($\sigma_{\text{per}}$ and $\sigma_{\text{par}}$), which are all incoming angle-dependent functions:

$$e_{\text{per}} = 0.993 - 1.76 \theta + 1.56 \theta^2 - 0.49 \theta^3$$  \hspace{1cm} (III-29)

$$e_{\text{par}} = 0.998 - 1.66 \theta + 2.11 \theta^2 - 0.67 \theta^3$$  \hspace{1cm} (III-30)

$$\sigma_{\text{per}} = -0.0005 + 0.62 \theta - 0.535 \theta^2 + 0.089 \theta^3$$  \hspace{1cm} (III-31)

$$\sigma_{\text{par}} = 2.15\theta - 5.02\theta^2 + 4.05\theta^3 - 1.085\theta^4$$  \hspace{1cm} (III-32)

It is pointed out that the above-mentioned particle rebound models were originally not available for CFX-4 users. The Forder model was built in the CFD-based erosion procedure by Edwards (2000). One important task of this research is to implement Grant and Tabakoff’s stochastic rebound model in the CFD-based erosion procedure and to investigate its effect on particle tracking and erosion pattern in elbows and plug tees.

**Erosion Modeling**

The third component of the CFD-based erosion prediction procedure is to calculate the erosion caused by sand particle impingements. In the particle tracking modeling section, particle impingement information such as impact speed, impact angle, and impact location as well as impingement intensity is stored. In this step, the sand particle impingement information is extracted and applied to appropriate erosion prediction equations to compute the erosion.

The quantity of erosion ratio can be determined if the particle impact speed and impact angle are known. The erosion ratio is defined as the amount of mass lost by the pipe wall due to particle impacts divided by the mass of particles impacting. The Other
quantity of interest is the penetration rate that is defined as the wall thickness loss due to sand particle erosion per unit time.

**Calculation of the Erosion Ratio**

There are several models available in literature that predict erosion rate. The relations developed by the E/CRC, such as the model for carbon steels by Ahlert (1994) and the model for aluminum by McLaury (1996), are applied to calculate the erosion rate as well as the penetration rate for this research. The models developed at the E/CRC for carbon steels and aluminum are described as functions that primarily depend on the particle impact speed and impact angle. These models are based on direct impingement tests at various angles and impingement velocities. In the model for carbon steels, the variables also include the Brinell hardness, $B$, of the wall material. According to Ahlert, the erosion ratio is given by

$$ ER = A F_s V^n F_0 $$

where $A$ is a material dependent coefficient; $F_s$ is a particle shape coefficient; $F_s = 1.0$ for sharp (angular), 0.53 for semi-rounded, or 0.2 for fully rounded sand particles; $n$ is an empirical constant; and $F_0$ is a function depending on the particle impact angle. The dependence on impingement angle, $\theta$, is given in Equation III-34 (Ahlert, 1994).

$$ F_0 = \begin{cases} a \theta^2 + b \theta & \text{for } \theta \leq \theta_0 \\ x \cos^2 \theta \sin(w \theta) + y \sin^2 \theta + z & \text{for } \theta > \theta_0 \end{cases} $$

where $\theta_0$ is an empirical angle constant. Table III-1 lists the empirical constants used in Equations III-33 and III-34 for Carbon Steel and Aluminum.
Table III-1. Erosion Model Empirical Constants.

<table>
<thead>
<tr>
<th>Empirical Constant</th>
<th>Material</th>
<th>Carbon Steel</th>
<th>Aluminum</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1559B⁻⁰·⁵⁹ × 10⁻⁹</td>
<td>2.388 × 10⁻⁷</td>
<td></td>
</tr>
<tr>
<td>θ₀</td>
<td>15°</td>
<td>10°</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>38.4</td>
<td>-34.79</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>22.7</td>
<td>12.3</td>
<td></td>
</tr>
<tr>
<td>w</td>
<td>1</td>
<td>5.205</td>
<td></td>
</tr>
<tr>
<td>x</td>
<td>3.147</td>
<td>0.147</td>
<td></td>
</tr>
<tr>
<td>y</td>
<td>0.3609</td>
<td>-0.745</td>
<td></td>
</tr>
<tr>
<td>z</td>
<td>2.532</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>1.73</td>
<td>1.73</td>
<td></td>
</tr>
</tbody>
</table>

**Calculation of the Penetration Rate**

The local penetration rate for the computational cell can be calculated once the mass loss of this computational cell is determined. In the last step of the erosion calculation, the penetration rate is computed for each cell on the wall, which allows the visualization of the penetration rate profile in the post-processor for the whole geometry surface. For each cell on the surface of the geometry, the local penetration rate is calculated by

\[
\dot{P}_j = \frac{\pi d_p^2}{6A_j} \left( \frac{\dot{N}_p}{N_p} \right) \left( \frac{\rho_p}{\rho_w} \right) \sum_i \text{ER}_{i,j} \tag{III-35}
\]

where \( A_j \) is the area of the local impinged computational cell face, \( \text{ER}_{i,j} \) is the local erosion ratio, \( \dot{N}_p \) is particle rate in particles per second flowing, and \( N_p \) is the total...
number of particles introduced in the flow. The subscript \( j \) refers to the computational cell of interest, and \( i \) denotes under consideration. Typically, \( N_p \) is a very large number, on the order of a hundred thousand, to minimize scatter and obtain a particle number independent erosion profile.

In this CFD-based erosion prediction procedure, one main assumption is made to calculate the wall mass loss as well as the wall penetration rate. It is assumed that the mass loss on the wall due to the particle impingement is evenly distributed on the computational cell surface where the impingement occurs. The approximation errors induced by this assumption can be minimized if the cell surface is sufficiently small and the number of simulated particles is large enough. A particle number sensitivity study is required to insure that the erosion predictions are not dependent on the number of particles simulated.
CHAPTER IV
VALIDATION OF FLOW MODELING IN SINGLE-PHASE FLOW

The CFD-based erosion prediction procedure possesses three components: flow modeling, particle tracking, and erosion calculation. This chapter discusses the validation of the turbulence models of single-phase flow. Flow simulations are performed in a 90 degree elbow and results are compared with experimental data available in literature (Enayet et al., 1982) in order to evaluate the accuracy of flow modeling.

The flow simulation in the elbow contains two tasks: grid refinement study and comparison of turbulence models. The first task is to conduct the sensitivity study of the flow solution to the grid of the elbow geometry and thus to determine an appropriate mesh that is employed in the second task. In the second task, different turbulence models are applied to simulate the flow in the elbow by using the appropriate mesh that is obtained from the first task.

Description of Problem

A flow model validation was performed on a 90º elbow geometry (Enayet et al., 1982) to verify the three-dimensional flow modeling capabilities. Enayet et al. (1982) employed a Laser-Doppler Velocimetry (LDV) technique and measured the flow through a 90-degree pipe bend. The turbulent flow measurements were taken with a Reynolds number of 43000. Velocity and turbulence intensity measurements were made in planes
perpendicular to the flow direction at \( x/D = -0.58 \) (upstream of bend); \( \beta = 30^\circ, 60^\circ, \) and \( 75^\circ \) (in the bend section); and \( x/D = 1.0 \) and \( 6.0 \) (downstream of bend), as shown in Figure IV-1. In-situ flow conditions in the elbow are provided in Table IV-1. The coordinate system of the elbow is defined as: \( x \) direction is the axial direction of the elbow; \( y \) direction is the normal direction from inside toward outside of the elbow; and \( z \) axis is the direction from the back to the front of the elbow. The coordinate system of the elbow is also demonstrated as Figure IV-2.

**Table IV-1. Flow Conditions in the Elbow (Enayet et al., 1982)**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipe diameter</td>
<td>48 mm</td>
</tr>
<tr>
<td>Turning radius ratio</td>
<td>2.8</td>
</tr>
<tr>
<td>Fluid bulk velocity</td>
<td>0.92 m/s</td>
</tr>
<tr>
<td>Fluid kinetic viscosity</td>
<td>0.01027 St</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>43,000</td>
</tr>
</tbody>
</table>
Figure IV-1. Nomenclature of Elbow 3-D Geometry and Measurement Stations.

Figure IV-2. The Coordinate System in the Elbow.
Grid Refinement Study

CFX-4 only uses structured grids for meshing. Thus a five-block geometry is built for the elbow along the axial direction, as shown in Figure IV-3a where the length ration of AC/AB is 0.48 and the mesh ratio of \( n1/n2 \) is 3/4, where \( n1 \) and \( n2 \) denote the grid number on AC and AB, respectively. Flow simulations are performed for four different meshes which are defined as Mesh 1, 2, 3, and 4 with the corresponding grid number on AC to be 3, 6, 9, and 12. There are 64, 256, 578, and 1024 cells on the cross section of the elbow for Mesh 1, 2, 3, and 4, respectively. The grid distribution of the pipe cross section of Mesh 3 is shown as Figure IV-3b. The inlet and outlet of the elbow is extruded to 10 diameters upstream and 15 diameters downstream, respectively. The grid number along the axial direction is 240 for all meshes.

(a). Five-block Pipe and Elbow.  
(b). Grid Distribution on the Cross Section.

Figure IV-3. Five-block Structured Elbow Geometry and a Sample Grid Distribution of the Cross Section.
The standard $k-\varepsilon$ turbulence model and the Quadratic Upwind differencing scheme (QUICK) are employed to simulate the flow field of the elbow for each mesh. The predicted $y^+$ value is 52 for these meshes. In order to examine the variation of the flow field with the grid, the comparisons of the predicted velocity profiles at different locations of the elbow were performed. Figures IV-4 and IV-5 show the predicted velocity profiles of each mesh and their comparison with the experimental data (Enayet et al., 1982) at the upstream plane ($x/D=-0.58$) and the downstream plane ($x/D=1.0$) along the y direction, respectively.

![Figure IV-4. Predicted Velocity Profiles of Different Meshes vs. Experimental Data (Enayet et al., 1982) at $x/D=-0.58$ in y Direction.](image-url)
As shown in Figures IV-4 and IV-5, the predicted velocity profile approaches a set of values and shows better agreement with the experimental data as the grid becomes finer. From Mesh 1 to 4, the variation of predicted velocity becomes smaller and it is negligible between Mesh 3 and Mesh 4. Thus it is deemed appropriate to use Mesh 3 as the representative mesh of the elbow. Further discussions of the flow field in the elbow predicted by different turbulence models are based on Mesh 3.

**Turbulence Modeling in the Elbow**

The turbulent flow in the elbow is predicted by applying the Differential Stress model, standard $k-\varepsilon$ model and RNG $k-\varepsilon$ model. The third-order Quadratic Upwind difference scheme is employed to simulate the turbulent flow. The predictions of velocity
and turbulent kinetic energy using different turbulent models were compared with experimental data (Enayet et al., 1982) at each measurement station as shown in Figure IV-1. The velocity has been normalized with respect to the average bulk velocity $U_{av}$ ($U_{av} = 0.92$ m/s) and the turbulence kinetic energy $k$ is converted to turbulence intensity $u'/U_{av}$, where $u'$ is the velocity fluctuation component and is determined by $k = \frac{3}{2}(u')^2$.

Both the normalized velocity and the turbulence intensity are plotted with respect to the dimensionless distance from the inside wall, $y/D$ or $z/D$. In addition, the fluctuation components of velocity predicted by the Reynolds Differential Stress model are also discussed.

**Comparison of Velocity Profiles**

Figures IV-6 and IV-7 show the predicted velocity profiles verses the measured velocity profile in the $y$ and $z$ directions, respectively, at the upstream station ($x/D = -0.58$), suggesting an axisymmetric inlet condition. Figures IV-8 to IV-10 contain the predicted and measured velocity profiles in the $y$ direction at three measurement stations in the elbow bend section ($\beta = 30^\circ$, 60$^\circ$, and 75$^\circ$), respectively. Comparisons of predicted velocity with experimental data in the $y$ direction at the downstream stations ($x/D = 1.0$ and $x/D = 6.0$) are shown in Figures IV-11 and IV-12, respectively.

In the upstream station ($x/D = -0.58$) the pipe flow is not yet affected by the elbow. From Figures IV-6 and IV-7, a good agreement of predicted velocity profiles with experimental data can be observed.

Due to the change of flow direction in the elbow, the inertia force of the fluid and the resulting centrifugal force interrupt the flow structure of straight pipe flow and cause
a difference of fluid velocity and field pressure between the inside wall and outside wall of the elbow, which causes the development of secondary flow in the form of a pair of vortices in the cross area of the elbow. At the inlet of the elbow, a relatively high velocity region forms on the inside of the elbow, but the secondary flow vortices displace the regions of relatively high velocity near the inside wall to the outside wall of the elbow and its downstream. The turbulence models are able to predict the high velocity region near the inside wall of the early section of the elbow, as shown Figure IV-8. Despite the complexity of the flow due to the secondary flow vortices, from Figure IV-9 to IV-12, results of flow simulations show reasonable agreement with the experimental data of Enayet et al. (1982).

Figure IV-6. Predicted Velocity Profiles Using Different Turbulence Models vs. Experimental Data (Enayet et al., 1982) at x/D = -0.58 in y Direction.
Figure IV-7. Predicted Velocity Profiles Using Different Turbulence Models vs. Experimental Data (Enayet et al., 1982) at x/D=-0.58 in z Direction.

Figure IV-8. Predicted Velocity Profiles Using Different Turbulence Models vs. Experimental Data (Enayet et al., 1982) at β = 30° in y Direction.
Figure IV-9. Predicted Velocity Profiles Using Different Turbulence Models vs. Experimental Data (Enayet et al., 1982) at $\beta = 60^\circ$ in $y$ Direction.

Figure IV-10. Predicted Velocity Profiles Using Different Turbulence Models vs. Experimental Data (Enayet et al., 1982) at $\beta = 75^\circ$ in $y$ Direction.
Figure IV-11. Predicted Velocity Profiles Using Different Turbulence Models vs. Experimental Data (Enayet et al., 1982) at x/D = 1.0 in y Direction.

Figure IV-12. Predicted Velocity Profiles Using Different Turbulence Models vs. Experimental Data (Enayet et al., 1982) at x/D = 6.0 in y Direction.
Comparison of Turbulent Kinetic Energy (Turbulence Intensity)

The comparisons of predicted turbulence intensity with experimental data are shown in Figures IV-13 to IV-19. From Figures IV-13 to IV-15, the predicted turbulence intensity profile in the y direction has good agreement with the experimental data in the upstream and early sections of the elbow. But the turbulence in the core region is overpredicted. Near the exit of the elbow and downstream, a high turbulence region occurs near the inside wall region due to the mixing of secondary flow vortices. Figures IV-16 to IV-18 show that the turbulence models under-predict the turbulence intensity in this region. Further downstream of the elbow, at station x/D = 6.0, the turbulence intensity decreases near the inside wall region when the strength of the secondary flow vortices decays due to energy dissipation. Increased accuracy of predicted turbulent kinetic energy profile is observed further downstream of the elbow, as shown in Figure IV-19.

Figure IV-13. Predicted Turbulence Intensity Using Different Turbulence Models vs. Experimental Data (Enayet et al., 1982) at x/D = -0.58 in y Direction.
Figure IV-14. Predicted Turbulence Intensity Using Different Turbulence Models vs. Experimental Data (Enayet et al., 1982) at $x/D = -0.58$ in $z$ Direction.

Figure IV-15. Predicted Turbulence Intensity Using Different Turbulence Models vs. Experimental Data (Enayet et al., 1982) at $\beta = 30^\circ$ in $y$ Direction.
Figure IV-16. Predicted Turbulence Intensity Using Different Turbulence Models vs. Experimental Data (Enayet et al., 1982) at $\beta = 60^\circ$ in $y$ Direction.

Figure IV-17. Predicted Turbulence Intensity Using Different Turbulence Models vs. Experimental Data (Enayet et al., 1982) at $\beta = 75^\circ$ in $y$ Direction.
Figure IV-18. Predicted Turbulence Intensity Using Different Turbulence Models vs. Experimental Data (Enayet et al., 1982) at x/D = 1.0 in y Direction.

Figure IV-19. Predicted Turbulence Intensity Using Different Turbulence Models vs. Experimental Data (Enayet et al., 1982) at x/D = 6.0 in y Direction.
**Velocity Fluctuations Predicted by the Reynolds Differential Stress Model**

The underlying assumption of Standard $k-\varepsilon$ model and RNG $k-\varepsilon$ model is that the turbulent flow is isotropic. With the cost of extra computational effort, the Reynolds Differential Stress model is able to compute the individual turbulent stress components. Figures IV-20 to IV-26 contain the profiles of the velocity fluctuations for three different directions predicted by the Reynolds Differential Stress model and the comparison with experimental data of Enayet *et al.* (1982) which is a mean value obtained using the measured turbulent kinetic energy.

Figures IV-20 and IV-21 show that, upstream of the elbow inlet, the fluctuations of the normal velocity ($u$) near the wall region are greater than those of the velocity in the radial direction ($v$ and $w$). Due to the axisymmetric feature of straight pipe flow, the fluctuation components of $v$ and $w$ are always identical. In the core region, the isotropic turbulent flow is predicted.

In the elbow section, the secondary flow vortices introduce extra variation for velocity fluctuation components in all directions, which results in the disappearance of isotropic turbulence in the core region. As demonstrated in Figures IV-22 to IV-26, depending on the strength of the secondary flow vortices, the turbulent fluctuation components varies in each direction in the elbow and downstream. From Figures IV-25 and IV-26, downstream of the elbow, the profile of predicted fluctuations of the main-stream direction velocity ($v$) possesses better agreement with the trend of the data of Enayet *et al.* (1982).
Figure IV-20. Predicted Turbulence Intensity by the Differential Stress Model vs. Experimental Data (Enayet et al., 1982) at $x/D = -0.58$ in $y$ Direction.

Figure IV-21. Predicted Turbulence Intensity by the Differential Stress Model vs. Experimental Data (Enayet et al., 1982) at $x/D = -0.58$ in $z$ Direction.
Figure IV-22. Predicted Turbulence Intensity by the Differential Stress Model vs. Experimental Data (Enayet et al., 1982) at $\beta = 30^\circ$ in $y$ Direction.

Figure IV-23. Predicted Turbulence Intensity by the Differential Stress Model vs. Experimental Data (Enayet et al., 1982) at $\beta = 60^\circ$ in $y$ Direction.
Figure IV-24. Predicted Turbulence Intensity by the Differential Stress Model vs. Experimental Data (Enayet et al., 1982) at $\beta = 75^\circ$ in $y$ Direction.

Figure IV-25. Predicted Turbulence Intensity by the Differential Stress Model vs. Experimental Data (Enayet et al., 1982) at $x/D = 1.0$ in $y$ Direction.
Figure IV-26. Predicted Turbulence Intensity by the Differential Stress Model vs. Experimental Data (Enayet et al., 1982) at x/D = 6.0 in y Direction.

Summary of Turbulence Modeling in the Elbow

The Reynolds Differential Stress model, Standard k–ε model, and RNG k–ε model were applied to simulate the turbulence flow in an elbow using the Quadratic Upwind (QUICK) difference scheme which is third-order accurate. Predicted velocity as well as turbulence intensity show reasonable agreement with the experimental results of Enayet et al. (1982). The flow in the elbow and downstream is characterized by the existence of secondary flow vortices which strongly affect the velocity profile as well as turbulence intensities. The results of the simulations show that the turbulent models employed are less accurate in predicting the flow field near the inside wall region of the elbow and downstream pipe where the vortices mix. More investigation of turbulence models as well as the flow in the elbow is necessary to understand the discrepancy.
between the simulations and the experimental data and to improve the prediction accuracy of turbulence models.

The assumption of isotropic turbulence is not appropriate for the cases where a sudden change of flow field exists. The Reynolds Differential Stress model has the potential to compute the flow turbulence more accurately than the $k-\varepsilon$ models.
CHAPTER V

EROSION PREDICTION IN ELBOWS AND PLUG TEES

This chapter is dedicated to investigating the sand erosion in plug tees and elbows under a variety of flow conditions of interest for single-phase flow by applying the CFD-based erosion prediction procedure which consists of three main steps: flow simulation, particle tracking, and erosion computation. During the simulations, the emphasis was on the influence of particle-wall interaction and rebound models on particle behavior and erosion patterns in the elbow and plug tee. The sensitivity of maximum erosion in the elbow and plug tee to the mesh of geometry and the number of particles simulated are studied. The difference of sand erosion between the elbows and plug tees for a wide range of flow conditions is also discussed. The simulation results of this chapter will be validated by the experimental erosion tests that are discussed in Chapter VI and be used to develop the simplified erosion prediction model for plug tees.

Effects of Particle Rebound Models

Sand erosion of the surface of the geometry is due to the sand particle impingements on the wall and the momentum transfer between the particle and the wall surface. Momentum loss of the particle accompanies with the impingement on the wall. The level of particle momentum loss during the impingement can be represented by
restitution coefficients that are the ratios of particle velocity components after impingement to the corresponding components before impingement, as demonstrated in Figure V-1 where $V_{1N}$, $V_{1T}$, $V_{2N}$, and $V_{2T}$ are the normal and tangential components of the particle incoming and reflected velocity, respectively, and $\theta$ is the particle impact angle. Particle-wall interaction has a profound impact on the behavior of particles upon impingement and thus affects the erosion pattern in the geometry.

\[ \begin{align*}
V_{1T} & \quad V_{2N} \\
V_{1N} & \quad \theta \\
V_{2T} &
\end{align*} \]

**Figure V-1. Schematic of Particle-Wall Interaction and Rebound.**

Researchers have shown that the particle incoming angle may have a significant effect on the restitution coefficients of both perpendicular and parallel components. Forder et al. (1998) proposed a particle-wall interaction and rebound model as functions of the particle incoming impact angle based on impingement testing using AISI 4130 carbon steel to determine the restitution coefficients, $e$. The restitution coefficient in the normal and tangential direction, $e_{\text{per}}$ and $e_{\text{par}}$, are determined by Equations V-1 and V-2, respectively.

\[ e_{\text{per}} = \frac{V_{2N}}{V_{1N}} = 1 - 0.4159 \theta + 0.5994 \theta^2 - 0.292 \theta^3 \]  
(V-1)
\[ e_{\text{par}} = \frac{V_{2T}}{V_{1T}} = 1 - 2.12 \theta + 3.077 \theta^2 - 1.1 \theta^3 \quad (V-2) \]

Grant and Tabakoff (1975), and Sommerfeld (1999) stated that particle rebound upon impingement on the wall is a stochastic event. Based on experimental data (for 2024 Aluminum Alloy and 200 µm sand particles), Grant and Tabakoff postulated the mean values of the coefficients of restitution (\( e_{\text{per}} \) and \( e_{\text{par}} \)). In addition, Grant and Tabakoff provided the standard deviations (\( \sigma_{\text{per}} \) and \( \sigma_{\text{par}} \)) of the restitution coefficients. Similar to the particle rebound model proposed by Forder et al. (1998), \( e_{\text{per}} \), \( e_{\text{par}} \), \( \sigma_{\text{per}} \) and \( \sigma_{\text{par}} \) of the stochastic particle rebound model proposed by Grant and Tabakoff are angle-dependent functions that are expressed in Equations V-3 to V-6.

\[ e_{\text{per}} = 0.993 - 1.76 \theta + 1.56 \theta^2 - 0.49 \theta^3 \quad (V-3) \]
\[ e_{\text{par}} = 0.998 - 1.66 \theta + 2.11 \theta^2 - 0.67 \theta^3 \quad (V-4) \]
\[ \sigma_{\text{per}} = -0.0005 + 0.62 \theta - 0.535 \theta^2 + 0.089 \theta^3 \quad (V-5) \]
\[ \sigma_{\text{par}} = 2.15 \theta - 5.02 \theta^2 + 4.05 \theta^3 - 1.085 \theta^4 \quad (V-6) \]

Particle trajectories in the flow field are determined by the flow of the carrier fluid, particle tracking model, and particle rebound model. In addition to the accurate prediction of the carrier flow field and particle-fluid interaction, the adoption of an appropriate particle rebound model is important to accurately predict the particle trajectories and thus obtain a statistically representative erosion pattern on the geometry surface. In order to examine the effect of particle rebound model on particle trajectories as well as erosion pattern, the stochastic particle rebound model of Grant and Tabakoff (1975) was implemented into the CFD-based erosion prediction procedure. The particle
rebound models proposed by Forder et al. (1998), and the stochastic particle rebound model were applied to predict the particle trajectories and compute the erosion in an elbow and plug tee, respectively. The flow conditions in the elbow and plug tee are summarized in Table V-1. As discussed in Chapter IV, the Reynolds Differential Stress model was applied to simulate the turbulent flow in the elbow and plug tee.

**Table V-1. Flow Conditions in the Elbow and Plug Tee.**

<table>
<thead>
<tr>
<th></th>
<th>Water</th>
<th>Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier Fluid</td>
<td>Water</td>
<td>Air</td>
</tr>
<tr>
<td>Velocity (ft/s)</td>
<td>20</td>
<td>150</td>
</tr>
<tr>
<td>Sand Volume Concentration (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sand Rate (lb/day)</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Pipe Diameter (inch)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Particle Diameter (µm)</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>Pipe Material</td>
<td>Carbon Steel</td>
<td></td>
</tr>
</tbody>
</table>

The particle trajectories predicted by the Forder (1998) particle rebound model and the stochastic particle rebound model of Grant and Tabakoff (1975) were examined by predicting 10 sample particles in the elbow and plug tee. These particles are released from a centerline of the inlet plane of the elbow and plug tee geometry and are place with equally spaced. The erosion pattern was predicted as well by simulating a large number of particles (100,000) randomly distributed at the inlet of the geometry of the elbow and plug tee for the conditions listed in Table V-1. The predicted sample particle trajectories and erosion pattern of the elbow and plug tee are discussed separately for the sample water case and the sample air case.
Sample Particle Trajectories and Erosion Patterns in Water

Figures V-2 and V-3 are particle trajectories in the elbow flowing water predicted by the Forder particle rebound model and the stochastic particle rebound model, respectively. Figures V-4 and V-5 contain the particle trajectories in the plug tee. Figures V-6 to V-9 show the erosion patterns in the elbow and plug tee predicted by these two particle rebound models.

In both the elbow and plug tee, predicted particle trajectories and erosion patterns for water using the Forder particle rebound model are extremely similar to particle trajectories and erosion patterns predicted by the stochastic particle rebound model, as shown in Figures V-2 to V-9. The particle rebound model affects the particle behavior by determining the reflected velocity after the particle impinges the wall. Due to high viscosity and high density of water, the drag force exerted on particles is dominant so that the influence of the particle rebound model on the particle trajectories as well as erosion pattern is negligible in statistical sense.

The high drag force exerted on the particles ensures that particles follow the flow streamlines of the carrier fluid (water) closely when particles are transported through the bend section of the elbow, as seen in Figures V-2 and V-3. Figures V-2 and V-3 also demonstrate that particles deviate from the flow streamlines of the fluid and approach the wall of the elbow at the exit region due to particle inertial force. Because the geometry transition and flow redirection in the plug tee are more dramatic than the elbow, Figures V-4 and V-5 show that more particles tend to deviate from the flow streamlines and impinge the wall at the joint corner of the plug tee rather than following the flow
streamlines when particles pass through the plug tee joint section. It is also possible that some particles recirculate in the plug section.

Figures V-6 and V-7 convey the information that the maximum erosion occurs around the exit region of the elbow. Two mechanisms contribute to this phenomenon. Firstly, as discussed in Chapter IV, the secondary flow vortices carry the fluid with high momentum formed in the inlet of the elbow toward the outer side of the elbow in the exit region. Thus the fluid velocity is high in this region. Secondly, the particle inertial force drives particles to deviate from fluid streamlines and approach to the wall, which results in localized particle impingements at the elbow exit region.

Figures V-8 and V-9 show that the maximum erosion is located at the joint corner of the plug tee. For this specific flow condition, the maximum erosion in the plug tee is about one point five times that of the elbow. In addition to the plug tee joint corner, a relative high erosion region is observed on the side of downstream pipe of the plug tee. Similar to the elbow, secondary flow exists in the plug tee and has a significant impact on the erosion pattern in the plug tee downstream section. In the plug section, the fluid is almost stagnant and particle velocities are very low. Erosion in this region should not be a concern although particle recirculation is observed.
Figure V-2. Particle Trajectories in Water Predicted by the Forder Particle Rebound Model in the Elbow.

Figure V-3. Particle Trajectories in Water Predicted by the Stochastic Particle Rebound Model in the Elbow.
Figure V-4. Particle Trajectories in Water Predicted by the Forder Particle Rebound Model in the Plug Tee.

Figure V-5. Particle Trajectories in Water Predicted by the Stochastic Particle Rebound Model in the Plug Tee.
Figure V-6. Erosion Pattern in Water Predicted by the Forder Particle Rebound Model in the Elbow.

Figure V-7. Erosion Pattern in Water Predicted by the Stochastic Particle Rebound Model in the Elbow.
Figure V-8. Erosion Pattern in Water Predicted by the Forder Particle Rebound Model in the Plug Tee.

Figure V-9. Erosion Pattern in Water Predicted by the Stochastic Particle Rebound Model in the Plug Tee.
Sample Particle Trajectories and Erosion Pattern in Air

Similarly, the particle trajectories and erosion patterns flowing air predicted by the Forder rebound model and the stochastic rebound model are discussed. Figures V-10 to V-13 show the particles trajectories and Figures V-14 to V-17 illustrate erosion patterns in the elbow and plug tee, respectively.

Particle inertia is dominant when the flow is redirected for the cases in which the carrier fluid possesses low density and low viscosity such as air. Figures V-10 and V-11 both show that all entrained particles impact the wall of the elbow corresponding to the projected area of elbow inlet. Upon the first impingement with the wall, most particles reflect downstream. Although in Figures V-10 and V-11 similar particle trajectories are predicted in the elbow using the Forder rebound model and the stochastic rebound model, the particle reflected speed and angle are slightly different predicted for the two particle rebound models. This difference is seen in the predicted erosion pattern in the elbow as well as maximum value of erosion rate. From Figures V-14 and V-15, the maximum erosion predicted by the Forder rebound model is 10.2 mil/lb that is about 20% greater than the stochastic rebound model. In Figure V-14, two antenna-like high erosion regions are observed. The antenna-like high erosion regions are due to the secondary impingements of the particles. Concentrated secondary impingements also contribute to an increase in the maximum erosion. This behavior did not occur when the stochastic particle rebound model was applied.

In the plug tee, all entrained particles also directly impinge the end surface of the plug tee. More importantly, most particles recirculate in the plug end section. Significant differences of predicted particle trajectories are observed between Figure V-12 and
Figure V-13. The sample particle trajectories in Figure V-12 using the Forder rebound model demonstrate that the particles become trapped in the plug section of the tee. Some individual particles impinge the wall in the plug section hundreds of times along a recirculation path before they go to the downstream section. Apparently, this simulation result is not physical because of the nature of the particle-wall collision rebound process. Particle recirculation and repeating impingements result in highly concentrated erosion in the plug tee, as illustrated in Figure V-16. The maximum erosion predicted by the Forder rebound model is 58 mil/lb. However, from experimental observations it is anticipated that the maximum erosion more uniformly occurs on the surface of the plug end. When simulations with the stochastic rebound were conducted, due to variations in particle rebounds introduced using the stochastic particle rebound model, the localized repeating impingements are not predicted although particle recirculation in the plug section is observed in Figure V-13. The randomly scattered particle rebound results in more evenly distributed particle impingements as well as erosion damage in the plug section. From Figure V-17, the maximum erosion is 3.9 mil/lb in the plug end of the plug tee.
Figure V-10. Particle Trajectories in Air Predicted by the Forder Particle Rebound Model in the Elbow.

Figure V-11. Particle Trajectories in Air Predicted by the Stochastic Particle Rebound Model in the Elbow.
Figure V-12. Particle Trajectories in Air Predicted by the Forder Particle Rebound Model in the Plug Tee.

Figure V-13. Particle Trajectories in Air Predicted by the Stochastic Particle Rebound Model in the Plug Tee.
Figure V-14. Erosion Pattern in Air Predicted by the Forder Particle Rebound Model in the Elbow.

Figure V-15. Erosion Pattern in Air Predicted by the Stochastic Particle Rebound Model in the Elbow.
Air Velocity = 150 ft/s
Erosion Rate (mil/lb)

Figure V-16. Erosion Pattern in Water Predicted by the Forder Particle Rebound Model in the Plug Tee.

Figure V-17. Erosion Pattern in Water Predicted by the Stochastic Particle Rebound Model in the Plug Tee.
Summary of Particle Rebound Model Effects

The simulations in water demonstrate that predictions based on the Forder particle rebound model and the stochastic particle rebound model are almost identical. For the cases where the carrier fluid has high density and viscosity and the flow velocity is relatively low, the fluid has a significant “drag” effect on the entrained solid particles. The slight difference of rebound speed and angle for individual particles has little effect on the overall erosion pattern and the maximum value of erosion.

In air, particle inertia plays a major role in determining particle trajectories when flow redirection occurs. Rebound speed and angle of particles that are determined by the particle rebound model will have a tremendous impact on particle trajectories and thus affect the erosion pattern when multiple impingements are expected. The simulations in the plug tee demonstrate that the application of the stochastic particle rebound model is required for the cases that may possess strong particle recirculation.

Sensitivity of Erosion to Geometry Mesh

In order to accurately predict the erosion rates, a grid sensitivity study must be performed. As an example, the grid refinement study for the air case in the elbow as listed in Table V-1 is demonstrated. During this grid refinement study, the stochastic particle rebound model was utilized, and 10,000 particles were released at the inlet of the elbow.

The grid refinement process was performed in two steps. The first step was to refine the grid on the plane that is normal to the flow direction with the axial grid spacing fixed. The second step is to adopt an appropriate mesh of the cross-area plane and to
refine the axial direction grid, especially for the elbow curved section. The schematic of the cross-area plane of the five-block structured elbow geometry is shown as Figure V-18(a). In Figure V-18(a), the length ratio AB/AC is equal to 2.4; N1 and N2 denote the grid numbers on line AC and AB, respectively. In the first step, the erosion prediction was performed for 9 different meshes. The grid number on length AB and AC for each mesh is listed in Table V-2. As a representative case, the cross-sectional view of Mesh No. 5 is presented in Figure V-18(b). For all these 9 meshes, the grid number of the elbow curved section (DE) is 30 where length DE and the mesh are shown in Figure V-19. In the second step of the grid refinement, the erosion is predicted for four different axial grid spacings with the cross-area plane grid spacing fixed. The axial grid numbers used on the elbow curved section (DE) are listed in Table V-3. It is worth noting that Mesh No. 5 of Table V-2 is equivalent to Mesh No. 11 of Table V-3.

Table V-2. Grid Numbers of the Cross-Sectional Plane Meshes.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>No. 1</th>
<th>No. 2</th>
<th>No. 3</th>
<th>No. 4</th>
<th>No. 5</th>
<th>No. 6</th>
<th>No. 7</th>
<th>No. 8</th>
<th>No. 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>16</td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>N2</td>
<td>5</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>24</td>
<td>30</td>
<td>36</td>
</tr>
</tbody>
</table>

Table V-3. Number of Grid in the Elbow Section Curve Meshes.

<table>
<thead>
<tr>
<th>Meshes</th>
<th>No. 10</th>
<th>No. 11 (No. 5)</th>
<th>No. 12</th>
<th>No. 13</th>
</tr>
</thead>
<tbody>
<tr>
<td>DE</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>60</td>
</tr>
</tbody>
</table>
(a). Schematic of Cross-Area Plane of the Five-block Elbow

(b). Representative Mesh of the Cross-Area Plane

Figure V-18. Cross-sectional View of the Five-block Elbow Geometry and Representative Mesh.

(a). Schematic of Elbow Curve Section on the Center Plane

(b). Representative Mesh of the Elbow Curve Section

Figure V-19. Front View of the Five-block Elbow Geometry and Representative Mesh on the Center Plane.
The predicted maximum erosion normalized using the coarsest grid (Mesh No. 1) of each mesh is shown as Figure V-20. In Figure V-20, the horizontal axis represents the cross-area plane grid density normalized with respect to the grid density of Mesh No. 1. From Figure V-20, the erosion approaches a specific value as the grid density increases. The variation of the maximum erosion between Mesh No. 8 and Mesh No. 9 is about 2%, and it is about 20% between Mesh No. 5 and Mesh No. 9. Twenty percent is well within the acceptable tolerance for erosion calculations. So Mesh No. 5 is used for further studies.

![Figure V-20. Variation of the Maximum Erosion with the Cross-Area Grid Density.](image)

As a combined consideration of prediction accuracy and computational cost, the cross-area plane Mesh No. 5 is selected in the second step. The variation of the normalized predicted maximum erosion for each mesh listed in Table V-3 is plotted as Figure V-21. The predicted maximum erosion of each mesh is normalized by that of
Mesh No. 10. The grid density of the elbow curved section using Mesh No. 10 is defined as 1. The erosion appears to converge with increases in elbow curved section axial grid density.

![Graph](image)

**Figure V-21. Variation of the Maximum Erosion with the Axial Direction Grid Density.**

From Figures V-20 and V-21, reasonable accuracy of erosion can be obtained from the predictions using Mesh No. 5 (or Mesh No. 11). The remaining simulations of this study adopted Mesh No. 5 for the elbow geometry. An identical grid spacing was used in the generation of the mesh of the plug tee.

**Particle Number Study**

For the particle tracking process of the CFD-based erosion prediction model, the user is required to specify the number of particles that are randomly released at the inlet of the geometry to represent the given mass of sand. In order to insure that a statistically
representative set of particle impingements are obtained, the effect of particle number on
the quantity of the predicted erosion needs to be investigated. The erosion prediction
procedure was applied to the elbow and the plug tee for six particle numbers: 1000, 5000,
10000, 25000, 50000, and 100000 for the air flow conditions listed at Table V-1. Figures
V-22 and V-23 demonstrate how the normalized predicted maximum erosion changes
with the input particle number in the elbow and the plug tee, respectively. The predicted
maximum erosion obtained for each particle number is normalized by erosion obtained
using 1000 particles in the elbow and the plug tee. Figures V-22 and V-23 demonstrate
that the effect of particle number on the predicted erosion is negligible when the particle
number is greater that 50000. Thus, 50000 particles were simulated at the inlet of the
elbow and plug tee for the remaining simulations of this study.

![Graph](image)

**Figure V-22.** Variation of the Predicted Maximum Erosion with the Number of
Particles in the Elbow.
Figure V-23. Variation of the Predicted Maximum Erosion with the Number of Particles in the Plug Tee.

Erosion Prediction in Elbows and Plug Tees

Erosion was predicted applying the CFD-based erosion prediction model for a broad range of flow conditions in elbows and plug tees. For these simulations, the variables include pipe diameter of elbow and plug tee, physical properties and velocity of carrier fluid, and particle diameter. The flow conditions are summarized as Table V-4. The simulation matrix (Table V-4) represents 240 simulation cases. For all cases, the turbulent flow in the elbow and plug tee was simulated using the Reynolds Differential Stress model, particle rebound velocity and angle was determined by the stochastic particle rebound model, and 50,000 particles were used to represent the sand mass flow rate which is 10 lb/day.
Table V-4. Simulation Matrix of Erosion Prediction in Elbows and Plug Tees.

<table>
<thead>
<tr>
<th>Pipe Diameter (inch)</th>
<th>1, 2, 4, 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity (ft/s)</td>
<td></td>
</tr>
<tr>
<td>Water</td>
<td>5, 10, 20</td>
</tr>
<tr>
<td>Air</td>
<td>50, 100, 150</td>
</tr>
<tr>
<td>Particle Diameter (µm)</td>
<td>50, 100, 150, 220, 300</td>
</tr>
</tbody>
</table>

In petroleum engineering practice, the cap (the end surface) of the plug tee can be made with a desired thickness. It is also more economical to maintain (i.e. make the end surface thicker) than other sections of the plug tee. Hence, two regions in the plug tee geometry are defined (as shown in Figure V-24) to clarify the erosion in corresponding regions: End Region and Side & Corner Region. Simulations show that little erosion occurs at the end region of plug tee for all water cases due to the presence of the almost stagnant fluid region. While for air cases, depending on flow conditions comparable erosion can be observed at both regions. All simulation results are listed in Appendix.

Figure V-24. End Region and Side & Corner Region of the Plug Tee.
In order to make direct comparisons of erosion between elbows and plug tees, the penetration ratio was introduced and is defined as the maximum penetration rate of the plug tee divided by the maximum penetration rate of the elbow. The relative penetration ratio of the end region as well as the side & corner region are determined by using this definition. The penetration ratio between plug tees (only applicable at Side & Corner region) and elbows for water cases are plotted in Figures V-25 to V-28 for different pipe diameter. Figures V-29 to V-32 and Figures V-33 to V-36 show the relative penetration ratio for air cases at the End region and Side & Corner region, respectively, for different pipe diameter.

Although the curves of relative penetration ratio behave somewhat differently in Figures V-25 to V-28, the overall trend is that the penetration ratio is greater than 1 for all flow conditions included. This means that in water plug tees experience more erosion than elbows. Previously, Figures V-2 to V-9 indicated that in water sand particles follow fluid streamlines closely in the elbow while in the plug tee more particles tend to deviate from the fluid and impinge the plug tee at the corner. It can be deduced from this study that it would not be reasonable to adopt the plug tee instead of elbow in an erosive environment where the fluid has high density and viscosity like water.

In Figures V-29 to 32 it is interesting to see that the penetration ratio at the End region is strongly affected by flow parameters including pipe diameter and particle diameter. Depending on flow parameters, a peak value of the relative penetration tends to appear. The trends of the penetration ratio also tend to converge to a constant of 0.5 as the particle diameter increases. As seen in Figures V-13 and V-17, particle recirculating in the plug section is the characteristic feature of particle behavior through the plug tee.
The potential consequence of particle recirculation in the plug section is concentrated erosion at a certain area of the plug tee. Detailed information of simulations tells that particle recirculation intensity is a matter of flow variables (velocity, pipe size and particle size), which is responsible for the variation of relative penetration ratio. When particle diameter reaches a certain value, particle recirculation becomes insignificant. This is the reason that the relative penetration ratio tends to become a constant having the value of 0.5. Similar trends of the penetration ratio of the Side & Corner region can be observed. In addition, the constant value of relative penetration ration is much less than in the End region. This information is very useful because more commonly the erosion occurring at this region determines the service life of plug tee.

![Graph](image)

**Figure V-25. Predicted Penetration Ratio between Plug Tee and Elbow with 1 inch Diameter for Water Cases.**
Figure V-26. Predicted Penetration Ratio between Plug Tee and Elbow with 2 inch Diameter for Water Cases.

Figure V-27. Predicted Penetration Ratio between Plug Tee and Elbow with 4 inch Diameter for Water Cases.
Figure V-28. Predicted Penetration Ratio between Plug Tee and Elbow with 8 inch Diameter for Water Cases.

Figure V-29. Predicted Penetration Ratio between Plug Tee (End Region) and Elbow with 1 inch Diameter for Air Cases.
Figure V-30. Predicted Penetration Ratio between Plug Tee (End Region) and Elbow with 2 inch Diameter for Air Cases.

Figure V-31. Predicted Penetration Ratio between Plug Tee (End Region) and Elbow with 4 inch Diameter for Air Cases.
Figure V-32. Predicted Penetration Ratio between Plug Tee (End Region) and Elbow with 8 inch Diameter for Air Cases.

Figure V-33. Predicted Penetration Ratio between Plug Tee (Side & Corner Region) and Elbow with 1 inch Diameter for Air Cases.
Figure V-34. Predicted Penetration Ratio between Plug Tee (Side & Corner Region) and Elbow with 2 inch Diameter for Air Cases.

Figure V-35. Predicted Penetration Ratio between Plug Tee (Side & Corner Region) and Elbow with 4 inch Diameter for Air Cases.
Figure V-36. Predicted Penetration Ratio between Plug Tee (Side & Corner Region) and Elbow with 8 inch Diameter for Air Cases.
CHAPTER VI

EXPERIMENTAL EROSION TESTS IN ELBOWS AND PLUG TEES AND

VALIDATION OF SIMULATION RESULTS

Erosion was predicted in elbows and plug tees for a wide range of flow conditions of interest by applying the CFD-based erosion prediction procedure. Results for predicted erosion in elbows as well as plug tees are summarized in Appendices 1 to 3. The relative penetration between elbows and plug tees is presented in Chapter V. These simulation results need to be validated. Thus this chapter is dedicated to the discussion of the experimental erosion tests conducted in elbows and plug tees as well as the validation of simulation results.

From Appendices 1 to 3 one can notice that the predicted erosion of water cases is less than air cases by 4 to 6 orders-of-magnitude. For two reasons the erosion of sand entrained in gas flow is a much greater concern than in liquid flow in practice: firstly, gas usually possesses much lower density and viscosity than liquid so that the drag on particles exerted by gas is much weaker than liquid; secondly, flow speed of gas is normally much larger than liquid. Accordingly, the focus of this chapter is on the validation of the simulations of air cases.
Experimental Erosion Tests in Air

Experiment Facility and Set-up

In order to validate the simulation results of the air cases, erosion tests were performed in test cells representing an elbow and a plug tee with 1-inch diameter. The schematic of the test flow loop is shown in Figure VI-1. The test cells and specimens of the elbow and plug tee are shown as Figures VI-2 and VI-3, respectively.

Figure VI-1. Schematic of the Air Test Flow Loop.
Figure VI-2. Test Cell and Specimen of the Elbow.

Figure VI-3. Test Cell and Specimens of the Plug Tee.
As shown in Figures VI-1 to VI-3, air with the injected sand flows in vertically upward and was discharged horizontally. The elbow specimen is a 0.25" × 0.25" bar that is bent to match the radius of curvature of the elbow. Specimen 1 of the plug tee covers the plug end surface. Specimens 2, 3, and 4 have widths of 0.25" and are placed on the center plane of the plug tee test cell. Elbow and plug tee specimens are made of aluminum.

**Experiment Procedure**

Simulation results of air cases illustrate that the predicted erosion is influenced by flow velocity, particle diameter, and pipe diameter. For the experimental set-up shown as Figure VI-1, the pipe diameter is 1 inch. In order to validate the applied CFD-based erosion model as well as the predicted penetration ratio between elbows and plug tees, experimental work consisting of two stages was conducted. The purpose of Stage I tests is to examine the trend of erosion in the elbow and plug tee as well as the penetration as a function of air velocity. While in Stage II tests, the particle size effects on the penetration ratio between elbows and plug tees are examined.

**Stage I Erosion Tests** In the Stage I tests, mass loss measurements of the elbow and plug tee specimens were taken for three air velocities (50 ft/s, 100 ft/s, and 150 ft/s) using Oklahoma No. 1 sand with a mean diameter of 150 µm (Li, 2003). For all the tests in this stage, the same mass of sand (1.1 lb) is injected into the flow loop through the sand injector during a fixed period of time (40 minutes). The test conditions of Stage I tests are listed in Table VI-1. Tests were repeated three times for each test condition.
Table VI-1. Conditions of Stage I Erosion Test.

<table>
<thead>
<tr>
<th>Pipe Diameter (inch)</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of Sand Injected (lb)</td>
<td>1.1</td>
</tr>
<tr>
<td>Test Time (min)</td>
<td>40</td>
</tr>
<tr>
<td>Particle Diameter (µm)</td>
<td>150</td>
</tr>
<tr>
<td>Fluid Velocity (ft/s)</td>
<td>50</td>
</tr>
<tr>
<td>Sand Volume Concentration (%)</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Using the measured mass loss, the average thickness losses of test specimens of the elbow and plug tee are calculated and the trends of erosion are then determined experimentally. In addition to the trend of erosion, the local erosion profile is also essential information to quantify the maximum thickness loss to compare with the penetration rate given by simulations. Hence, local thickness loss profiles were measured using a profilometer (shown in Figure VI-4). The profilometer is a piece of equipment that measures profiles of the material surface. The local thickness loss is determined by comparing two surface profiles obtained before and after the erosion test at the identical location. With this method, a reference surface is required. Deep and narrow “V”-shaped scratches are drawn on the specimen surface. The cross-sectional profile (depth and shape) of the scratches are also recorded by the profilometer. It is assumed that no particle reaches the bottom of the scratches so that the bottom is not eroded. Thus the bottoms of the scratches serve as the profilometer measurement reference locations. Surface profiles are highly location sensitive. In order to gain a meaningful comparison of surface profiles before and after the erosion test, it is critical to ensure that the surface
profiles are taken at the identical location. Two scratches (major scratch and minor scratch) with a certain angle are introduced on the specimen surface for each profilometer measurement. The surface profile is taken across these the two scratches and the profilometer reading line is perpendicular to the major scratch. The distance between two scratches along the measurement (or reading line) direction is thus uniquely determined, which provides an effective means to identify the location where the surface profile is taken before and after the erosion test. A difference in the distance between two scratches obtained before and after the test is an indicator to the deviation of the reading line location after the test from the one before the test. This difference can be minimized by slightly adjusting the reading location of the profilometer reading rod.

Figure VI-4. Profilometer and Surface Profile Measurement.

As will be discussed in the following sub-section, the measured mass losses of specimens 3 and 4 of the plug tee are less than the elbow specimen and specimens 1 and 2 of the plug tee by an order of magnitude. Therefore, the erosion of specimens 3 and 4 is
considered negligible with respect to other specimens. Profilometer measurements are only made on the elbow specimen and specimens 1 and 2 of the plug tee. On the elbow specimen, profilometer measurements are taken at seven locations in order to capture the maximum thickness loss. Measurement locations start at 20° downstream of the elbow inlet and are made in 10° increments along the elbow specimen curve. The measurement locations and corresponding scratches are demonstrated in Figure VI-5. A profilometer measurement is taken along the center line of specimen 1 of the plug tee from the bottom toward the top. For specimen 2, measurements are taken along the center line for both edges. The reading line direction is from the edge toward the corner of specimen 2. Measurement locations and scratches of specimens 1 and 2 of the plug tee are shown in Figure VI-6.

![Figure VI-5. Measurement Locations and Scratches of the Elbow Specimen.](image)
Figure VI-6. Measurement Locations and Scratches of Specimens 1 and 2 of the Plug Tee.

For profilometer measurements, a high air velocity (150 ft/s) is used in order to get measurable thickness losses from specimens during a reasonable period of time and reasonable amount of sand injected. Table VI-2 includes the relative parameters of these measurements.

**Table VI-2. Test Conditions of Profilometer Measurements of Stage I Tests.**

<table>
<thead>
<tr>
<th></th>
<th>Elbow</th>
<th>Plug Tee</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of Sand Injected (lb)</td>
<td>1.1</td>
<td>6.6</td>
</tr>
<tr>
<td>Test Time (min)</td>
<td>40</td>
<td>240</td>
</tr>
<tr>
<td>Pipe Diameter (inch)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Particle Diameter (µm)</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>Fluid Velocity (ft/s)</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>Sand Volume Concentration (%)</td>
<td>0.00025</td>
<td></td>
</tr>
</tbody>
</table>
**Stage II Erosion Tests**  
Experiment for Stage II tests are to validate the simulated penetration ratio between elbows and plug tees from another aspect: particle size effect. Different from Stage I tests, the sand particle diameter changes while the air velocity stays constant in this test stage. Three size groups of sand are sieved from Oklahoma No. 1 sand. They are 50~90, 150~170, and 212~250 µm. The air velocity is 150 ft/s for all tests. The test conditions of Stage II tests are summarized in Table VI-3. For all conditions listed in Table VI-3, mass loss measurements as well as local thickness measurements (profilometer measurements) are conducted for both the elbow and plug tee specimens. Each test is repeated three times for mass loss measurements. For local thickness loss measurements, the measurement locations as well as scratches on the surface of specimens of the elbow and plug tee are the same as shown in Figures VI-5 and VI-6.

### Table VI-3. Conditions of Stage II Tests.

<table>
<thead>
<tr>
<th>Pipe Diameter (inch)</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of Sand Injected (lb)</td>
<td>2.2</td>
</tr>
<tr>
<td>Test Time (min)</td>
<td>80</td>
</tr>
<tr>
<td>Fluid Velocity (ft/s)</td>
<td>150</td>
</tr>
<tr>
<td>Sand Volume Concentration (%)</td>
<td>0.00025</td>
</tr>
<tr>
<td>Particle Diameter (µm)</td>
<td>50~90</td>
</tr>
</tbody>
</table>

**Experimental Data**

This sub-section analyzes the experimental data obtained from Stage I and II tests.
**Stage I Erosion Tests**  
Measured mass losses of specimens of the elbow and plug tee for conditions listed in Table VI-1 are gathered in Table VI-4. As clearly shown in Table VI-4, measured mass losses of specimens 3 and 4 of the plug tee are negligible as compared to the elbow specimen and specimens 1 and 2 of the plug tee. Thus, specimens 3 and 4 are excluded from later discussions. The average mass loss with 95% confidence intervals of the elbow specimen as well as specimens 1 and 2 of the plug tee are plotted in Figure VI-7 versus air velocity. The experimental data is very repeatable.

In the last column of Table VI-4, the average thickness losses of the specimens are estimated according to the corresponding average mass loss. The average thickness loss of the elbow specimen is calculated by dividing the average mass loss by the specimen projected area from the elbow inlet direction. The average thickness loss of specimen 1 of the plug tee is obtained when the average mass loss is assumed to be distributed evenly over the whole specimen surface. The average thickness loss of specimen 2 is estimated similarly. By divided by the mass of sand that passes through the test cell, the calculated average thickness loss is finally given in units of mil/lb which means the thickness loss (mil) due to unit mass of sand (lb).
Table VI-4. Measured Mass Loss of Specimens from Test Stage I.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Velocity (ft/s)</th>
<th>Mass Loss (g)</th>
<th></th>
<th></th>
<th>Average Thickness Loss (mil/lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Test 1</td>
<td>Test 2</td>
<td>Test 3</td>
<td>Average</td>
</tr>
<tr>
<td>Elbow</td>
<td>50</td>
<td>0.0021</td>
<td>0.0021</td>
<td>0.0021</td>
<td>0.0020</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.0056</td>
<td>0.0048</td>
<td>0.0049</td>
<td>0.0051</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>0.0011</td>
<td>0.0087</td>
<td>0.0094</td>
<td>0.0094</td>
</tr>
<tr>
<td>Plug Tee Specimen 1</td>
<td>50</td>
<td>0.0014</td>
<td>0.0015</td>
<td>0.0012</td>
<td>0.00133</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.0055</td>
<td>0.0061</td>
<td>0.0060</td>
<td>0.00583</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>0.0103</td>
<td>0.0116</td>
<td>0.0075</td>
<td>0.0098</td>
</tr>
<tr>
<td>Plug Tee Specimen 2</td>
<td>50</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0006</td>
<td>0.00053</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.0014</td>
<td>0.001</td>
<td>0.0013</td>
<td>0.00123</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>0.002</td>
<td>0.002</td>
<td>0.0015</td>
<td>0.00183</td>
</tr>
<tr>
<td>Plug Tee Specimen 3</td>
<td>50</td>
<td>0.0002</td>
<td>0.00</td>
<td>0.0004</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.0003</td>
<td>0.004</td>
<td>0.0005</td>
<td>0.0004</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>0.0009</td>
<td>0.0005</td>
<td>0.0001</td>
<td>0.0005</td>
</tr>
<tr>
<td>Plug Tee Specimen 4</td>
<td>50</td>
<td>0.000</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.0001</td>
<td>0.000</td>
<td>0.0002</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>0.0004</td>
<td>0.0001</td>
<td>0.0003</td>
<td>0.0003</td>
</tr>
</tbody>
</table>
For the profilometer measurements, surface profiles of the elbow and plug tee specimens are recorded before and after the tests. With the reference of the scratch bottom, the differences between the profiles before and after test are determined as the local thickness loss of the specimen. For the elbow specimen, the local thickness losses at seven reading locations are examined and the maximum thickness loss occurs at reading location No. 5. Figure VI-8 illustrates the local thickness loss along the reading line at reading location No. 5 of the elbow specimen. The maximum thickness loss of specimen 1 occurs at the beginning of the reading line which is close to the bottom of the plug tee as shown in Figure VI-6(a). As for specimen 2, greater thickness loss is detected at the discharge side and the maximum thickness loss is located at the end of the reading line which corresponds to the plug tee corner. Similarly, Figures VI-9 and VI-10 plot the representative surface profiles of specimens 1 and 2 of the plug tee, respectively.
Local Thickness Loss at Reading Location No. 5 of the Elbow Specimen

Maximum Thickness Loss = 34 µm

Figure VI-8. Local Thickness Loss at Reading Location No. 5 and the Maximum Thickness Loss of the Elbow Specimen.

Local Thickness Loss along the Reading Line of Specimen 1 (micron)

Maximum Thickness Loss = 120 µm

Figure VI-9. Local Thickness Loss along the Reading Line and the Maximum Thickness Loss of Specimen 1 of the Plug Tee.
From Figure VI-8 the maximum thickness loss of the elbow specimen is 34 µm, which results in the maximum penetration of 1.22 mil/lb. According to Figure VI-9, the maximum thickness loss of specimen 1 is 120 µm and the corresponding maximum penetration is 0.72 mil/lb. As seen in Figure VI-10, the maximum thickness loss of specimen 2 is 30 µm which is equivalent to the penetration of 0.12 mil/lb. It is worth mentioning that the profilemeter measurements on the plug tee were repeated once. Also identical results were gained from these two tests.

**Stage II Erosion Tests**

Collected experimental data of Stage II tests for flow conditions shown in Table VI-3 are listed in Table VI-5. The average thickness loss is determined by the same manner as discussed in Stage I tests. In addition, Table VI-5
contains the maximum thickness loss of the elbow and plug tee specimens that is obtained from the profilometer measurements.

**Table VI-5. Mass Loss and Thickness Loss of Specimens from Stage II Tests.**

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Particle Diameter (µm)</th>
<th>Mass Loss (g)</th>
<th>Thickness Loss (mil/lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Test 1</td>
<td>Test 2</td>
</tr>
<tr>
<td>Elbow</td>
<td>50~90</td>
<td>0.0261</td>
<td>0.0288</td>
</tr>
<tr>
<td></td>
<td>150~170</td>
<td>0.0243</td>
<td>0.0182</td>
</tr>
<tr>
<td></td>
<td>212~250</td>
<td>0.0161</td>
<td>0.0188</td>
</tr>
<tr>
<td>Plug Tee Specimen 1</td>
<td>50~90</td>
<td>0.0285</td>
<td>0.0243</td>
</tr>
<tr>
<td></td>
<td>150~170</td>
<td>0.0230</td>
<td>0.0200</td>
</tr>
<tr>
<td></td>
<td>212~250</td>
<td>0.0120</td>
<td>0.0135</td>
</tr>
<tr>
<td>Plug Tee Specimen 2</td>
<td>50~90</td>
<td>0.0110</td>
<td>0.0100</td>
</tr>
<tr>
<td></td>
<td>150~170</td>
<td>0.0024</td>
<td>0.0027</td>
</tr>
<tr>
<td></td>
<td>212~250</td>
<td>0.0022</td>
<td>0.0025</td>
</tr>
</tbody>
</table>

**Validation of Simulation Results**

This section applies experimental erosion data collected from the 1-inch flow loop to evaluate the predicted erosion in the elbow as well as plug tee for air cases. Validations are conducted from three views: erosion pattern, velocity effect, and particle size effect.

**Erosion Pattern**

For (Stage I tests) profilometer measurements, local thickness losses of the elbow specimen at seven locations are detected. The maximum thickness loss occurs at
measurement location No. 5. This observation agrees with predicted erosion pattern. Along the elbow curve on the center plane, erosion is extracted from simulation results. Predicted erosion and experimental erosion data of elbow are plotted in Figure VI-11. In this diagram, 0° represents elbow inlet and 90° is the elbow outlet. As seen in Figure VI-11, the erosion pattern of the elbow is well predicted. This diagram also indicates that the applied erosion prediction model is over-predictive.

![Figure VI-11. Predicted and Measured Erosion Profile of the Elbow.](image)

From the profilometer measurement of the plug tee test, the plug end surface (specimen 1) is found to have the maximum thickness loss which is about 0.72 mil/lb. Figure VI-9 indicates that the maximum erosion occurs at the very beginning section of the reading line. As for specimen 2, the high erosion region is at the corner section where the two pipes meet. This information confirms the erosion pattern predicted by the
stochastic particle rebound model (such as seen in Figure V-17). In order to further validate the significance of applying the stochastic particle rebound model, the measured and predicted erosion profiles of the plug tee end surface along the center line are compared in Figure VI-12. Both measurements and predictions give a reasonable distribution of erosion on the plug tee end surface rather than the highly concentrated erosion predicted by the Forder particle rebound model which is not stochastic. Again, Figure VI-12 shows that the erosion results form the model are much higher than the data.

**Figure VI-12. Predicted and Measured Erosion Profile of the Plug Tee End Surface.**

**Erosion versus Velocity**

For flow conditions in Table VI-1, erosion rates are predicted for the elbow and plug tee. In addition to the predicted maximum erosion, average erosion rate is also
estimated for locations that correspond to the elbow specimen, specimens 1 and 2 of the plug tee. The predicted results are compared with the experimental data attained from Stage I tests, as demonstrated by Figures VI-13 to VI-15. From these diagrams, one can notice that the trend of erosion both in the elbow and plug tee is well predicted by the CFD-based erosion prediction model although the erosion is over-predicted. But more importantly, the over-prediction is consistent and repeatable. Figures VI-13 through VI-15 indicate that the CFD-based erosion prediction model is reliable in predicting the trend of erosion. Meanwhile, the over-prediction of erosion suggests that refinement of the erosion calculation model (particularly Equation III-32) is necessary in order to improve the performance of the CFD-based erosion prediction model. This work is being performed in a separate study of the Erosion/Corrosion Research Center.

In addition to the absolute quantity of erosion in both the elbow and plug tee, another important aspect of this research is to look at the penetration ratio between the plug tee (End Region, and Side & Corner Region) and the elbow. Figures VI-16 and VI-17 serve to compare the predicted and measured penetration ratios between both regions of the plug tee and the elbow. The comparison of predicted and measured average penetration ratios shows that the trend of penetration ratio between the plug tee and the elbow can be reasonably predicted. Meanwhile, the quantity of measured maximum penetration ratio (from the data of profilometer measurement) is accurately predicted. The comparison between the predictions and experimental data concludes that although erosion in both elbows and plug tees is over-predicted, the penetration ration between the plug tee and the elbow is reasonably predicted.
Figure VI-13. Predicted and Measured Erosion at the Elbow versus Air Velocity.

Figure VI-14. Predicted and Measured Erosion at the Plug Tee End Region versus Air Velocity.
Figure VI-15. Predicted and Measured Erosion at the Plug Tee Side & Corner Region versus Air Velocity.

Figure VI-16. Predicted and Measured Penetration Ratio at the Plug Tee End Region versus Air Velocity.
Particle Size Effect

Simulation results reveal that the relative penetration ratio between the plug tee (both the End Region and Side & Corner Region) and the elbow is also strongly affected by particle diameter. Experimental erosion Stage II tests are performed to validate this observation from simulations. Figures VI-18 and VI-19 are diagrams that compare the predicted and measured penetration ratios.

In Figure VI-18, experimental data (both the measured maximum and average penetration ratios) indicates that the particle size has little effect on the penetration ratio between the plug tee End Region and the elbow. The predicted trend shows that the penetration ratio decreases with particle size and becomes a constant when particle diameter is big enough. For relatively large particle sizes (greater than 150 µm), the trend and the quantity of the relative penetration ratio is accurately predicted. While for smaller
particle sizes, the relative penetration ratio is over-predicted. Careful analysis of the simulation information (including erosion pattern, particle deposition rate and particle impingement intensity) indicates that particle recirculation intensity becomes stronger with decreasing sand size, which results in more concentrated maximum erosion in the End Region of the plug tee. This is the reason why the erosion is more uniformly distributed when particle diameter increases. Thus, the penetration ratio for small particle sizes is over-predicted because of numerical exaggeration of particle recirculation in the plug section of the plug tee. The comparison in Figure VI-18 and the discussion are based on the experimental data and simulation results obtained from the plug tee and elbow with 1-inch diameter. From Figures V-28 to V-32 that plot the relative penetration ratio for different pipe diameters, although the trends are irregular for small particle sizes, one can observe that it converges to become a constant which has the value of about 0.5 when particle diameter is big enough.

As seen in Figure VI-19, a similar trend of the penetration ratio at the Side & Corner Region as the End Region of the plug tee is predicted. Excellent agreement between the experimental data and simulations is observed in Figure VI-19. At this point one question arises, why does the over-prediction of the penetration ratio for small sand sizes at the End Region shown in Figure VI-18 not appear in Figure VI-19. The predicted erosion patterns for related flow conditions are examined. The predicted maximum erosion at the Side & Corner Region is located exactly at the center of the corner, which is consistent with experiment results. It can be assumed that the majority of erosion at the corner is caused by the particle initial impingement and the impingement of particles leaving the plug section. Thus, the predicted trend presented in Figure VI-19 is free of the
numerical discrepancy that is included in Figure VI-18. Again, it is pointed out that this discussion is based on the numerical and experimental results and observation of the erosion obtained in the plug tee and elbow with 1-inch diameter and an air velocity of 150 ft/s. In fact, as shown in Figure V-33, the trends of the penetration ratio for different air velocities are similar to each other. However, Figures V-34 to V-36 demonstrate that, depending on the actual flow condition, the trend of the penetration ratio may not monotonically decrease with particle diameter. From the erosion pattern analysis, under some conditions the maximum erosion at the Side & Corner Region occurs at the plug section rather than the corner. Exaggerated particle recirculation causes the over-predicted erosion not only at the End Region but also at the plug section of the Side & Corner Region, which results in irregular behavior of the trend of penetration ratio as shown in Figures V-30 to V-32 and Figures V-34 to 36.

![Graph of Predicted and Measured Penetration Ratio vs Particle Diameter](image)

**Figure VI-18. Predicted and Measured Penetration Ratio at the Plug Tee End Region versus Particle Diameter.**
Further Experimental Tests

Experimental tests for Stage I and II previously discussed in the elbow and plug tee are conducted at low sand volume concentrations. For these tests, much greater mass loss and maximum thickness loss of the plug tee are measured at the plug tee end surface (Specimen 1) than the other test specimens placed in the plug tee. The erosion of Specimen 1 of the plug tee is comparable to the erosion of the elbow is the general impression one can gain from the analysis of the experimental data of Stage I and II tests.

Erosion of gas/sand flow in the elbow and plug tee at high sand concentrations and high air velocities was investigated by Bourgoyne (1989) in a 2-inch diameter flow system. Bourgoyne data shows that the erosion in the plug tees is about two-orders of magnitude less than the erosion in the elbow under similar conditions. In addition, high
erosion occurred at the exit of the plug tee rather than the plug end surface. Table VI-6 gives the test conditions and typical results obtained by Bourgoyne. Figure VI-20 shows the high erosion region observed by Bourgoyne.

**Table VI-6. Erosion in the Elbow and Plug Tee for High Sand Volume Concentration Reported by Bourgoyne (1989).**

<table>
<thead>
<tr>
<th></th>
<th>Air Velocity (m/s)</th>
<th>Sand Rate (kg/s)</th>
<th>Sand Rate (m³/s)</th>
<th>Sand Volume Concentration (%)</th>
<th>Erosion (m/s)</th>
<th>Erosion (mil/lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Elbow</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>111</td>
<td>0.55</td>
<td>2.08e-4</td>
<td>0.0925</td>
<td>6.15e-5</td>
<td>1.98</td>
</tr>
<tr>
<td></td>
<td>141</td>
<td>0.198</td>
<td>7.46e-5</td>
<td>0.0261</td>
<td>4.10e-5</td>
<td>3.67</td>
</tr>
<tr>
<td></td>
<td>141</td>
<td>0.081</td>
<td>3.06e-5</td>
<td>0.0107</td>
<td>1.55e-5</td>
<td>3.39</td>
</tr>
<tr>
<td><strong>Plug Tee</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>127</td>
<td>0.35</td>
<td>1.32e-4</td>
<td>0.0513</td>
<td>5.57e-7</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>141</td>
<td>0.34</td>
<td>1.46e-4</td>
<td>0.0511</td>
<td>4.73e-7</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>141</td>
<td>0.06</td>
<td>2.27e-5</td>
<td>0.0079</td>
<td>3.81e-7</td>
<td>0.11</td>
</tr>
</tbody>
</table>

**Figure VI-20. The High Erosion Region in the Plug Tee with High Sand Volume Concentrations Observed by Bourgoyne (1989).**
To shed more light on erosion behavior in the plug tee and the penetration ratio between plug tees and elbows, erosion tests are performed with comparable sand volume concentrations of the tests conducted by Bourgoyne. Tests are repeated 3 times for each condition. Mass loss measurements were taken for specimens of the elbow and plug tee. Table VI-7 summarizes the test results and the flow conditions under which tests are taken. In Table VI-7 the experimental data is converted to average thickness loss in mil/lb. Table VI-7 informs that Specimen 3 of the plug tee (see Figure VI-3) picks up measurable mass loss. The test conditions and average data of Stage I tests are represented in Table VI-8 to facilitate the comparison of the experimental results obtained from low and high sand volume concentration erosion tests.

Table VI-7. Erosion Tests in the Elbow and Plug Tee with High Sand Volume Concentration.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Velocity (m/s)</th>
<th>Sand Rate (kg/s)</th>
<th>Sand Volume Concentration (%)</th>
<th>Average Thickness Loss (mil/lb)</th>
<th>Penetration Ratio (Plug Tee/Elbow)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elbow</td>
<td>34</td>
<td>0.011</td>
<td>0.0244</td>
<td>0.116</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.011</td>
<td>0.0166</td>
<td>0.412</td>
<td></td>
</tr>
<tr>
<td>Plug Tee Specimen 1</td>
<td>34</td>
<td>0.011</td>
<td>0.0244</td>
<td>0.0013</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.011</td>
<td>0.0166</td>
<td>0.0038</td>
<td>0.010</td>
</tr>
<tr>
<td>Plug Tee Specimen 2</td>
<td>34</td>
<td>0.011</td>
<td>0.0244</td>
<td>0.0038</td>
<td>0.033</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.011</td>
<td>0.0166</td>
<td>0.013</td>
<td>0.033</td>
</tr>
<tr>
<td>Plug Tee Specimen 3</td>
<td>34</td>
<td>0.011</td>
<td>0.0244</td>
<td>0.032</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.011</td>
<td>0.0166</td>
<td>0.011</td>
<td>0.028</td>
</tr>
</tbody>
</table>
Table VI-8. Erosion Tests in the Elbow and Plug Tee with Low Sand Volume Concentration (Stage I).

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Velocity (m/s)</th>
<th>Sand Rate (kg/s)</th>
<th>Sand Volume Concentration (%)</th>
<th>Average Thickness Loss (mil/lb)</th>
<th>Relative Erosion Ratio (Plug Tee/Elbow)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elbow</td>
<td>34</td>
<td>2.08e-4</td>
<td>0.0005</td>
<td>0.408</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>2.08e-4</td>
<td>0.00025</td>
<td>0.752</td>
<td></td>
</tr>
<tr>
<td>Plug Tee Specimen 1</td>
<td>34</td>
<td>2.08e-4</td>
<td>0.0005</td>
<td>0.15</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>2.08e-4</td>
<td>0.00025</td>
<td>0.25</td>
<td>0.33</td>
</tr>
<tr>
<td>Plug Tee Specimen 2</td>
<td>34</td>
<td>2.08e-4</td>
<td>0.0005</td>
<td>0.072</td>
<td>0.077</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>2.08e-4</td>
<td>0.00025</td>
<td>0.107</td>
<td>0.062</td>
</tr>
</tbody>
</table>

According to Tables VI-7 and VI-8, the penetration ratio of Specimen 1 of the plug tee to the elbow for high sand rates has a significant reduction with respect to low sand rates. The penetration ratio of Specimen 2 also decreases by about 50%. Specimen 3, on which the erosion is negligible in Stages I and II tests, has comparable thickness loss to Specimen 2. Relatively, high erosion of the plug tee occurs on Specimens 2 and 3 rather than Specimen 1 as shown by Stage I tests. This preliminary study of erosion pattern with high sand concentration in plug tees agrees with Bourgoyne’s (1989) observations.

In fact, the underlying mechanism of sand erosion in plug tees in high sand volume concentration is extremely complicated. As seen in Figure VI-21, experimental data in Table VI-7 suggests that with high sand volume concentration sand interference occurs in the plug section due to the interaction of reflected particles from the plug end surface with incoming sand particles. This zone of interference covers the plug end
surface to protect it from erosion. The particle interaction effect also dominates at the joint section of the two pipes where the incoming particles collide with the outgoing particles from the plug section due to flow recirculation and particle reflection, which also contributes to the decreased number of particles that impinge the wall of plug end surface. Because of the difference in the geometry of the elbow and plug tee, particle interaction is far less strong and the sand interference does not exist in the elbow, which results in much less erosion in the plug tee in high sand volume concentration.

In depth experimental and numerical investigations of particle interaction for high sand rates are much needed to develop a better understanding of the erosion problem for such conditions.

Figure VI-21. Schematic of Sand Interference and Particles Interaction in the Plug Tee for High Sand Volume Concentration.
Summary

Experimental erosion tests are conducted in a 1-inch elbow and plug tee for both low and high sand volume concentrations. Sand concentration has a tremendous impact on the erosion pattern of the plug tee as well as the penetration ratio between the plug tee and the elbow. At low sand concentrations, the plug tee experiences erosion that has the same order of magnitude as the elbow. The erosion of the plug tee can be negligibly small with respect to the elbow when sand concentration is relatively high.

Being on the conservative side, the focus of this research is on the erosion in low sand volume concentrations. Experimental erosion tests for Stages I and II are assigned to validate the applied CFD-based erosion prediction model and simulation results obtained for similar flow conditions. The trend of erosion in both the elbow and plug tee versus air velocity is reasonably predicted although the absolute quantity of erosion is over-predicted. Refinement of the erosion calculation model in Equation III-32 is necessary in order to accurately calculate the erosion. The erosion pattern predicted by the stochastic particle rebound model (Grant and Tabakoff, 1975) is also consistent with experimental observations. Meanwhile, reasonable agreement is achieved between the measured and predicted penetration ratio versus air velocity.

The predicted trend of penetration ratio versus particle size is also validated. At the Side & Corner Region, good agreement is shown between the predictions and the measurements. At the End Region, due to numerically exaggerated particle recirculation in the plug section, the relative penetration ratio is over-predicted for small particle sizes. Appropriate adjustment of simulation results presented in Figures V-28 to V-36 is required in the development of the simplified erosion model for plug tees.
CHAPTER VII

DEVELOPMENT OF THE SIMPLIFIED EROSION PREDICTION MODEL FOR PLUG TEES

Following the discussion of simulation results, experimental data, and their comparison presented in Chapter V and Chapter VI, this chapter is assigned to develop a simplified model to predict the erosion of plug tees in air. From previous discussions, it is noted that this simplified model is to calculate the penetration ratio between plug tees (in both the End Region and Side & Corner Region) and elbows. This model will be implemented into SPPS. With this simplified model and established predictive model for elbows implemented in SPPS, the maximum erosion in plug tees can be conveniently determined.

Model Development

Good agreement between the predicted and measured penetration ratios in Side & the Corner Region of the plug tee is seen in Figure VI-19. However, discrepancies exist between the predictions and measurements obtained in the End Region of the plug tee, as shown in Figure VI-18. Figure VI-18 suggests that adjustment of the simulation results presented in Figures V-29 to V-36 is required to develop the mechanistic erosion model.
**End Region of Plug Tees**

In Figure VI-18, the penetration ratio is over-predicted for the End Region of the plug tee for small particle sizes. As seen in Figure VI-12, the erosion pattern on the plug tee end surface is well predicted when the particle size is 150 µm. Figure VII-1 gives the information about the detailed comparison of the measured and predicted erosion profiles for small particle size.

![Diagram showing predicted and measured erosion on the plug end surface](image)

**Figure VII-1. Predicted and Measured Erosion Profile on the Plug End Surface for Small Particle Diameter (70 µm).**

Figure VII-1 shows that when the particle diameter reduces to 70 µm the predicted erosion along the center line of the plug tee end surface varies dramatically, while a more evenly distributed erosion profile is measured. Particle tracking in the (1-
inch) plug tee demonstrates that particle recirculation within the plug section is strengthened when particle diameter decreases, which results in a more localized erosion distribution on the end surface of the plug tee. It can be deduced from Figures VI-18 and VII-1 that particle recirculation within the plug section is numerically exaggerated. The numerical exaggeration results in concentrated erosion the plug tee end surface, which causes over-predicted penetration ratio in this area.

![Comparison of Experimental Data with Penetration Ratio Predicted by the Stochastic and Non-stochastic Particle Rebound Model](image)

Figure VII-2. Comparison of Experimental Data with Penetration Ratio Predicted by the Stochastic and Non-stochastic Particle Rebound Model.

Although the trends of penetration ratio behave differently in Figures V-29 to V-32, they share a common feature that the penetration ratio is a constant value of about 0.5 when the particle size is sufficiently large. While experimental results (Figure VI-18) indicate that the relative penetration ratio is nearly independent of particle size and that it has the value around 0.5. Analysis of particle tracking and predicted erosion patterns for
large particles explains that particle recirculation is not significant and erosion is reasonably evenly distributed. This is the reason penetration ratio is a constant for big particles. For smaller particles, exaggerated particle recirculation contributes to the irregularity of the trend of penetration ratio. Summarily, based on analysis of numerical and experimental results, it is recommended that a value of penetration ratio between the plug tee End Region and the elbow of 0.5 be used.

**Side & Corner Region of Plug Tees**

As compared to Figure VI-18, the penetration ratio between the plug tee and the elbow shown in Figure VI-19 demonstrates a better agreement between the predictions and the data for small particle sizes. The key lies in the predicted erosion pattern for the corresponding small particle size (70 µm). As seen in Figure VII-3(a), the maximum erosion of the Side & Corner Region of the plug tee occurs around the plug joint corner. It is obvious that the erosion at the joint corner is not dominated by particle recirculation as strongly as the plug end surface because the majority of particle recirculation occurs within the plug section. Thus particle recirculation contributes less to the penetration ratio at the corner than the plug end surface.

Similar to Figures V-30 through V-32, the trend of penetration ratios shown in Figures V-34 through V-36 between the plug tee Side & Corner Region and the elbow are not monotonically related to particle diameter. The reason is that for some flow conditions the predicted maximum erosion of the Side & Corner Region of the plug tee is detected in the plug section. For instance, the erosion pattern for one specific case (air velocity is 150 ft/s; pipe diameter is 2 inch; and particle diameter is 50 µm) is demonstrated in Figure VII-3(b). For these cases, the erosion of the Side & Corner
Region is dominated by particle recirculation as strongly as the plug end surface. Numerical discrepancy of particle recirculation results in the over-predicted penetration ratio in the plug section (for Side & Corner Region) as well.

![Erosion Patterns at Side & Corner Region of the Plug Tee](image)

**Figure VII-3. Erosion Patterns at Side & Corner Region of the Plug Tee.**

The previous discussion concludes that the penetration ratio of the plug tee Side & Corner Region can be strongly affected by over-predicted particle recirculation. In order to accurately determine the penetration ratio, it is essential to distinguish the contribution of the first impingement of a particle and particle recirculation to the penetration ratio. Accordingly, the penetration ratio that is due to the particle first impingement only between the plug tee (End Region and Side & Corner Region) and the elbow is determined, as presented in Figures VII-4 to VII-11. The important feature of Figures VII-4 to VII-11 is that the relative penetration ratio due to particle first impingement appears to be a function of particle diameter. The predicted penetration
ratio at both the End Region and Side & Corner Region due only to particle recirculation is calculated by applying the values in Figures V-29 to V-36 then subtracting the values in Figures VII-4 to VII-11. Figures VII-4 to VII-7 confirm that the penetration ratio of the End Region approaches the recommended constant value of 0.5 as the particle diameter increases, which agrees with the previous discussion about the relative penetration ratio in the End Region of the plug tee. In order to effectively include the particle recirculation effect as well as avoid its exaggerated portion, an empirical particle recirculation impingement correction factor is introduced. This factor is used to correct the over-predicted particle recirculation erosion for the Side & Corner Region. The difference between the penetration ratio only due to particle’s first impact at the End Region in Figures VII-4 to VII-11 and 0.5 demonstrates that particle recirculation impingements contribute to erosion. This difference divided by the erosion only due to particle recirculation yields the correction factor. The determination of recirculation compensation factor can be described as follows:

\[ 0.5 = \text{First Impingement} + \text{Recirculation} \times \text{Correction Factor} \quad (\text{VII-1}) \]

Thus, the original simulation results for the Side & Corner Region of the plug tee (shown in Figures V-33 to V-36) are corrected by applying the empirical particle recirculation correction factor for identical flow conditions, the first impingement values (shown in Figures VII-8 to VII-11) and corresponding particle recirculation impingement. This process can be expressed as Equation VII-2:

\[ \text{Corrected Erosion} = \text{First Impingement} + \text{Recirculation} \times \text{Correction Factor} \quad (\text{VII-2}) \]

The corrected penetration ratios using Equation VII-2 are shown in Figures VII-12 to VII-15. Comparisons between Figures VII-12 to VII-15 and Figures VII-8 to VII-11
show that the compensation of particle recirculation in the plug tee Side & Corner Region does not alter the general trend of relative penetration ratio demonstrated in Figures VII-8 to VII-11. In Figures VII-12 to VII-15, the variation of the trends of penetration ratio obtained for different velocities is negligibly small. Hence, the mean value of the penetration ratio for different velocities for a specific particle size and pipe diameter is used as the representative value for that given condition. Figure VII-16 summarizes the trend of penetration ratio between the plug tee Side & Corner Region and the elbow varying the pipe diameter as well as particle diameter. The simplified erosion model for the plug tee Side & Corner Region is based on these ratios.

![Graph showing predicted penetration ratio between plug tee (end region) and elbow with 1 inch diameter due to particle first impingement.](image)

**Figure VII-4.** Predicted Penetration Ratio between Plug Tee (End Region) and Elbow with 1 inch Diameter due to Particle First Impingement.
Figure VII-5. Predicted Penetration Ratio between Plug Tee (End Region) and Elbow with 2 inch Diameter due to Particle First Impingement.

Figure VII-6. Predicted Penetration Ratio between Plug Tee (End Region) and Elbow with 4 inch Diameter due to Particle First Impingement.
Figure VII-7. Predicted Penetration Ratio between Plug Tee (End Region) and Elbow with 8 inch Diameter due to Particle First Impingement.

Figure VII-8. Predicted Penetration Ratio between Plug Tee (Side & Corner Region) and Elbow with 1 inch Diameter due to Particle First Impingement.
Figure VII-9. Predicted Penetration Rate between Plug Tee (Side & Corner Region) and Elbow with 2 inch Diameter due to Particle First Impingement.

Figure VII-10. Predicted Penetration Ratio between Plug Tee (Side & Corner Region) and Elbow with 4 inch Diameter due to Particle First Impingement.
Figure VII-11. Predicted Penetration Ratio between Plug Tee (Side & Corner Region) and Elbow with 8 inch Diameter due to Particle First Impingement.

Figure VII-12. Corrected Penetration Ratio between Plug Tee (Side & Corner Region) and Elbow with 1 inch Diameter.
Figure VII-13. Corrected Penetration Ratio between Plug Tee (Side & Corner Region) and Elbow with 2 inch Diameter.

Figure VII-14. Corrected Penetration Ratio between Plug Tee (Side & Corner Region) and Elbow with 4 inch Diameter.
Figure VII-15. Corrected Penetration Ratio between Plug Tee (Side & Corner Region) and Elbow with 8 inch Diameter.

Figure VII-16. Variation of Penetration Ratio between Plug Tee (Side & Corner Region) and Elbow with Pipe Diameter and Particle Diameter.
The curves in Figure VII-16 are correlated in a dimensionless formulation that is given by Equation VII-3:

\[
MPR = \begin{cases} 
- \left( 0.0526 \ln \frac{L_0}{d_p} + 0.4404 \right) \ln \left( \ln \frac{10^6 d_p}{D} \right) + 3.6964 \left( \frac{L_0}{d_p} \right)^{-0.0852} & d_p < 65 \, \mu m \\
- \left( 0.0994 \ln \frac{L_0}{d_p} + 0.1054 \right) \ln \left( \ln \frac{10^6 d_p}{D} \right) + 4.0 \times 10^{-5} \frac{L_0}{d_p} + 1.9268 & d_p \geq 65 \, \mu m 
\end{cases}
\]

(VII-3)

where \( MPR \) is the maximum penetration ratio between the plug tee Side & Corner Region and the elbow; \( L_0 \, (m) \) is the distance for a particle to reach its terminal velocity in air; \( d_p \, (\mu m) \) denotes particle diameter; and \( D \, (m) \) represents pipe diameter. The predictions using this correlation are compared with CFD predictions as well as experimental, which are presented in Figure VII-17.
As presented in Figure VII-17, the accuracy of the simplified model is evaluated by comparing the predictions of the simplified model with CFD predictions as well as experimental data. The simplified model agrees with CFD predictions extremely well. Meanwhile, both the mechanistic model and CFD model prediction have reasonable agreement with experimental data.

**Summary**

Based on numerical simulations and experimental erosion data obtained for the plug tee and elbow, a simplified erosion predictive model is developed to calculate the maximum penetration ratio between the plug tee (End Region and Side & Corner Region) and the elbow.

Particle recirculation is the dominant feature in the plug tee. Erosion of the plug tee is due to two mechanisms: the first impact of a particle and subsequent impingements resulting from particle recirculation. Although simulation results have been improved significantly by applying the stochastic particle rebound model, particle recirculation is still numerically exaggerated for small particle sizes. It is recommended that the maximum penetration ratio between the End Region of the plug tee and the elbow be assigned a value of 0.5. As for the Side & Corner Region of the plug tee, it is essential to distinguish the contribution of the particle first impingement and particle recirculation. In order to reflect the feature of particle recirculation in the plug tee as well as to avoid the numerical discrepancies of particle recirculation, an empirical particle recirculation correction factor is introduced. This factor is determined from impingement information (both first and recirculation) on the End Region, and it is applied to the Side & Corner
Region for the identical flow conditions. Then the maximum penetration ratio of the Side & Corner Region is determined by summing the first impingement and impingements from particle recirculation that are weighted by the particle recirculation correction factor. With these results, a simplified model is developed for the Side & Corner Region of the plug tee. This model is in a dimensionless form which is a function of flow characteristic lengths (pipe diameter, particle diameter, and the distance for a particle to reach terminal velocity).
CHAPTER VIII
MULTIPHASE FLOW MODELING

As discussed in Chapter II, due to the inherent complexity of multiphase flows, from a physical as well as numerical point of view, “general” applicable computational fluid dynamics (CFD) codes are non-existent. Nevertheless, various multiphase flow CFD models have been developed. These models can be divided in two categories: Eulerian-Lagrangian model, and Eulerian-Eulerian model. In general, the Eulerian-Lagrangian approach is suitable for continuous-dispersed flows. The Two-fluid (or Multi-fluid) model is a typical multiphase flow model using the Eulerian-Eulerian approach.

Multi-Fluid Model

The multiphase model implemented in CFX 4 is the multi-fluid model. This model can also be simplified as homogenous or mixture model. For the multi-fluid model the generic scalar advection-diffusion equation takes the following form (CFX-4 Solver Manual, 2000):

\[
\frac{\partial}{\partial t} (r_a \rho_a \Phi_a) + \nabla \cdot (r_a \rho_a U_a \Phi_a - \Gamma_a \nabla \Phi_a)) = r_a S_a + \sum_{\beta=1}^{M_a} c_{ap} (\Phi_\beta - \Phi_a) + \sum_{\beta=1}^{M_a} (m_{ap} \Phi_\beta - m_{pa} \Phi_a)
\]

(VIII-1)
Note:

1. Phases are labeled by Greek indices $\alpha$, $\beta$; $M_p$ denotes the number of phases; and $r_\alpha$ denotes the volume fraction of each phase;

2. The term $c_{\alpha\beta}(\Phi_\beta - \Phi_\alpha)$ describes inter-phase transfer of $\Phi$ between phases $\alpha$ and $\beta$;

3. $c_{\alpha\alpha} = 0$, $c_{\alpha\beta} = c_{\beta\alpha}$. Hence the sum over all phases of inter-phase transfer terms in zero;

4. The term $(\dot{m}_{\alpha\beta} \Phi_\beta - \dot{m}_{\beta\alpha} \Phi_\alpha)$ only arises if inter-phase mass transfer takes place. $\dot{m}_{\alpha\beta}$ is the mass flow rate per unit volume into phase $\alpha$ from $\beta$.

The mass and momentum conservation equations for no heat or mass transfer can be derived as Equations VIII-2 and VIII-3, respectively:

\[
\frac{\partial}{\partial t}(r_\alpha \rho_\alpha) + \nabla \cdot (r_\alpha \rho_\alpha U_\alpha) = 0 \quad \text{(VIII-2)}
\]

\[
\frac{\partial}{\partial t}(r_\alpha \rho_\alpha \Phi_\alpha) + \nabla \cdot \left( r_\alpha \left( \rho_\alpha U_\alpha \otimes U_\alpha - \mu_\alpha \left( \nabla U_\alpha + (\nabla U_\alpha)^T \right) \right) \right)
= r_\alpha (B - \nabla P_\alpha) + \sum_{\beta=1}^{N_p} c_{\alpha\beta} (U_\beta - U_\alpha) + F_\alpha \quad \text{(VIII-3)}
\]

where $B$ is the body force and $F_\alpha$ denotes the interfacial non-drag terms including the virtual mass force, lift force, wall lubrication force, and turbulent dispersion force.

**Inter-phase Drag** The total drag force per unit volume exerted on particles (or bubbles, droplets) with diameter $d$ is given by Equation VIII-4:

\[
D_{\alpha\beta} = \frac{3}{4} \frac{C_p}{d} r_\beta \rho_\beta |U_\beta - U_\alpha| (U_\beta - U_\alpha) \quad \text{(VIII-4)}
\]
where \( C_d \) is the drag coefficient. Various correlations (Schiller and Nauman, 1933, Ishii and Zuber, 1979, etc.) have been formulated to calculate \( C_d \). Thus, the inter-phase drag term \( c_{(d)}^{\alpha\beta} \) has the form:

\[
c_{(d)}^{\alpha\beta} = \frac{3}{4} \frac{C_d}{d} r_{\beta} \rho_a |U_\beta - U_a|
\]  
(VIII-5)

**Virtual Mass Force**  
This force is given in terms of the relative accelerations of the phases:

\[
F_\alpha = r_{\beta} \rho_a C_{VM} \left( \frac{D_\beta U_\beta}{Dt} - \frac{D_a U_a}{Dt} \right) 
\]  
(VIII-6)

where \( C_{VM} \) is a particle shape dependent constant, being 0.5 for individual spherical particles;

**Lift Force**  
This force is given in terms of slip velocity and the curl of the continuous phase velocity by:

\[
F_\alpha = r_{\beta} \rho_a C_L \left( U_\beta - U_a \right) \times \nabla \times U_a 
\]  
(VIII-7)

where \( C_L \) can take values between 0.01 and 0.05 for viscous flow;

**Wall Lubrication Force**  
This force is in the normal direction away from a wall and decays with the distance. This force is determined by:

\[
F_\alpha = \frac{r_{\beta} \rho_a (U_\beta - U_a)^2}{d} \max \left[ 0, \left( C_1 + C_2 \frac{d}{y_m} \right) \right] \hat{n} 
\]  
(VIII-8)

where \( y_m \) is the distance to the wall and \( \hat{n} \) is the normal to the wall. Typically, \( C_1 \) and \( C_2 \) takes a value of -0.01 and 0.05, respectively. This means the force only exists in the region less than 5 times of the particle (or bubble, droplet) diameter from the wall. This force can therefore only be seen for sufficiently fine grids;
**Turbulent Dispersion Force**

This force depends on the amount of turbulence in the continuous phase and the gradient of the volume fraction:

\[ F_a = -C_{TD} \rho_a k_a \nabla \alpha \]  

(VIII-9)

where \( C_{TD} \) is so-called turbulent dispersion coefficient and can take a value around 0.1.

**Multiphase Flow Modeling**

This section will be dedicated to modeling multiphase bubbly flow, annular flow and slug flow by employing the multi-fluid model available in CFX 4. Using the multi-fluid model that accounts for inter-phase transfer, the phase that has greatest density is normally identified as the primary (continuous) phase; other phases are then secondary (dispersed) phases with appropriately assigned mean bubble or particle diameter. The other important variable is the drag force model. In this research, the Ishii-Zuber (1979) model is employed.

**Bubbly Flow**

Air/water bubbly flow was modeled in a vertical upward square channel with a blockage (Tapucu, 1988) and results were compared with the experimental data collected by Tapucu. The square channel geometry has a cross section of 12.65 mm × 12.65 mm. A blockage with the dimension of 2.66 mm × 3.22 mm × 12.65 mm is placed at 0.6 m downstream of the channel inlet. The square channel and blockage are schematically demonstrated in Figure VIII-1. Two different flow conditions reported by Tapucu (1988) were studied in this channel. Table VIII-1 contains the summary of flow conditions.
Table VIII-1. Flow Conditions of Air/water Bubbly Flow in Square Channel.

<table>
<thead>
<tr>
<th></th>
<th>Air Mass Flux (kg/m²s)</th>
<th>Water Mass Flux (kg/m²s)</th>
<th>Air Volume Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.5</td>
<td>1512.5</td>
<td>19</td>
</tr>
<tr>
<td>Case 2</td>
<td>1.8</td>
<td>1528.0</td>
<td>40</td>
</tr>
</tbody>
</table>

Figure VIII-1. Schematic of the Square Channel and Blockage Geometry.

The bubbly flow in the channel is turbulent. Although multiple turbulence models are available, as the CFX-4 Solver Manual recommends only the Standard k–ε was tested in this study. The Sato (1975) model is used in CFX to take the bubble induced turbulence into account. The third-order Quadratic Upwind (QUICK) differencing scheme was selected to solve the governing equations. According to Winterton et al. (2001), the mean air bubble diameter in the channel takes the value of 0.6 mm (5% of pipe diameter).
The predicted air volume fraction as well as pressure distribution in the vertical direction are compared with the experimental data of Tapucu (1988). Although a 3D model was used in the simulations, only average data is available along the channel axis. Hence, the average quantities of the parameters of interest were taken for the cross-sectional area at the desired locations along the channel from the 3D simulation results. Figures VIII-2 to VIII-5 show the predicted average local air void fraction as well as pressure distribution in the channel axial direction compared with experimental data.

As seen in Figures VIII-2 to VIII-5, good agreement of predicted air void fraction as well as pressure distribution with experimental data for both cases is obtained. It is obvious that multi-fluid multiphase model is able to provide acceptable simulation results for bubbly flow.

![Graph](image)

**Figure VIII-2.** Comparison of Predicted Average Axial Air Void Fraction with Experimental Data (Tapucu *et al.*, 1988) of Case 1.
Figure VIII-3. Comparison of Predicted Average Axial Pressure Distribution with Experimental Data (Tapucu et al., 1988) of Case 1.

Figure VIII-4. Comparison of Predicted Average Axial Air Void Fraction with Experimental Data (Tapucu et al., 1988) of Case 2.
Uprising Taylor Bubble

The original attempt of this section was to simulate the vertical upward air/water slug pipe flow which is featured by an uprising bullet-shaped elongated bubble (also known as Taylor bubble) surrounded by water (as seen in Figure VIII-6) in every slug unit. In general the Taylor bubble is traveling at a velocity that is different from the surrounding liquid, which means that slug flow is essentially a transient multiphase flow phenomenon. The length of the Taylor bubble can be more than ten times to several hundred times the pipe diameter. A typical pipe length to simulate a slug flow is in the order of a thousand times the pipe diameter. To accomplish this modeling task, a supercomputer installed with a parallelized CFD code is required. Unfortunately, such a powerful facility is currently not available at the Erosion/Corrosion Center (E/CRC).
Davis and Taylor (1950) found that elongated bubbles are formed when air passes through the vertical pipe filled with water. This phenomenon is similar to slug flow to a great extent. Instead of simulating a slug flow, the flow of air rising in stagnant water is simulated in this study to demonstrate the capability of CFX-4 in modeling the Taylor bubble. The model is set up as follows (shown as Figure VIII-7): the flow field is a two-domain straight pipe with diameter of 1 inch; the exit of the pipe is open to atmosphere and the bottom and side wall are defined as wall boundaries; water and air are initially assigned to fill the upper and lower domain, respectively.

In fact, the stated model set-up is very numerically difficult at the beginning of the simulations because of the large difference between the fluid properties across a very thin boundary between the gas and liquid phases. The solver has some difficulties within the first few time steps of the simulation. It was also found that the simulation process is extremely sensitive to several parameters including time step, assigned mean diameter of air bubbles, under-relaxation factor, and boundary conditions. Within the first few time
steps, very small values of time step and under-relaxation factor are highly recommended (0.001 s and 0.1, respectively). In the beginning, the model is in a quasi-stable status although the water tends to replace the air in the lower domain. Simulations crash while simply applying the model demonstrated in Figure VIII-7. It seems that an external force that acts as a fluctuation is desired to trigger the simulation process. Therefore, the bottom wall boundary condition is modified as a velocity inlet boundary that allows air to enter with a small velocity (0.1 m/s). After two time steps, the incoming air is shut off and the bottom becomes wall. It will also be appropriate to have a small air bubble mean diameter (1 mm) for the first several time steps. Smaller bubbles tend to mix better with water, which helps break up the equilibrium between the water and air and thus facilitate the start of the simulation.

Figure VIII-7. Boundary and Initial Conditions of the Two-domain Model Set-up.
The primary goal of this model is to capture the formulation and rising of the Taylor bubble. Hence the interested variables are the volume fraction of water and air at different time steps. Figure VIII-8 illustrates the development and rising of the Taylor bubble in the stagnant water in terms of the air void fraction at different time steps. In Figure VIII-8(a) only the air domain and boundary with the water domain is demonstrated. The dimension scale is the same for Figures VIII-8(b) to 8(f). In Figures VIII-8(g) and 8(h), different dimension scales are used in order to include the complete Taylor bubble as well as the bottom of the pipe. In addition, the velocity vectors of water around the Taylor bubble head and tail are shown in Figure VIII-9 at the same time step as Figure VIII-8(f).
Figure VIII-8. The Development and Rising of Taylor Bubble in Stagnant Water.
Figure VIII-8. The Development and Rising of Taylor Bubble in Stagnant Water.

Figure VIII-9. Water Velocity Distribution around the Taylor Bubble.
Figure VIII-8(a) shows the initial status of the model set-up. Figures VIII-8(b) to VIII-8(d) show the intermediate status of the Taylor bubble during development. As seen in Figure VIII-8(f), at $t = 0.5$ s a bullet-shaped Taylor bubble is completely formed because it is surrounded by the liquid film between the bubble and the wall as well as the liquid at the bottom of the pipe. The Taylor bubble begins to rise in the pipe after this time step. In front of the Taylor bubble head there exists a transition region which is also referred as a smeared interface. The smeared interface is usually caused by numerical diffusion and can be minimized by reducing the time step and grid size. But these two parameters need to be well incorporated. A trailing tail of the Taylor bubble is also observed and water is entrained in the Taylor bubble from the open region between the trailing tails. In reality the Taylor bubble is more usually followed by a mixing zone (also referred as slug body). It is suggested that this model can be improved by two means: introducing initial velocity fluctuations three-dimensionally to break the tail; and using a 3-fluid model (air/water/dispersed air bubble) to enforce the mixing of water and dispersed air bubbles. Overall, the development and rising of the Taylor bubble in stagnant water is well predicted by the multi-fluid model of CFX.

As Figure VIII-9(a) shows, water sledded toward the wall by the uprising Taylor bubble squeezes between the Taylor bubble and wall and has a negative velocity (downward flow). As seen in Figure VIII-9(b), water is dragged by the Taylor bubble and tends to be mixed with the Taylor bubble tail.

**Annular Flow**

The flow field of annular flow consists of the continuous liquid film on the wall, the continuous gas core flow with possible entrained liquid droplets, as shown in Figure
For annular flow modeling, the analysis of forces on the liquid film is of vital importance. These forces include the wall shear, the interfacial shear at the film/gas interface, and the force of gravity. In vertical channels, all these forces act along the main flow direction. In addition, there are lateral forces maintaining the liquid film on the wall. The modeling of lateral forces becomes important for horizontal or inclined annular flows, where they govern the film thickness distribution along the channel perimeter. In vertical flows, once the annular flow regime is established, the net effect of lateral forces is very small as compared to the axial forces and is mainly reflected in maintaining the film structure.

Antal et al. (1998) stated that non-drag forces resulting from mechanisms such as lift, dispersion, wall effects, and turbulence produce a net lateral force on the liquid-gas interface. Antal et al. categorized these forces as collective forces that drive liquid to the wall and maintain the film and dispersive forces that tend to break the film to form droplets. For stabilized fully developed vertical annular flow, collective forces and dispersive forces are in quasi-equilibrium such that the lateral force can be ignored. Collective forces are greater than dispersive forces while the annular flow film developing.

According to this analysis, the multi-fluid model described by Equations VIII-2 to VIII-9 can be used to simulate fully developed annular flow but does not lead to the formation of annular liquid film on the wall. This explains that although extensive effort has been made in this study to model the upward vertical annular flow by applying the currently available multi-fluid model in CFX, no converged solution was achieved.
If a fully developed annular flow is assumed, the liquid film thickness as well as film velocity can be calculated by coupling the liquid phase and gas phase flow field. The liquid film is forced on the wall and occupies the corresponding cell space that is determined by applying mass and momentum equations. The liquid-gas interface is assigned as the wall of the gas phase which can include entrained droplets and be solved by the multi-fluid model. Phases are coupled via the interfacial shear stress. This approach has been successfully applied by Antal et al. (2000) who predicted the liquid film thickness of circular annular pipe flow with heat transfer. It is noted that this method requires tremendous numerical implementation and further discussion is beyond the scope of this research.
Summary

This section briefly reviewed various multiphase flow CFD models and summarized the available multi-fluid model in CFX 4. This multi-fluid model was applied to simulate bubbly flow in a square channel with a blockage, Taylor bubble rising in stagnant water, and annular pipe flow. Simulations show that the multi-fluid model has promising potential in providing acceptable results for bubbly flow and predicting the formation of Taylor bubbles. However, model modifications are necessary for annular flow modeling.
CHAPTER IX
EROSION PREDICTION IN MULTIPHASE FLOW

In multiphase flow, penetration and adhesion are two major mechanisms when sand particles interact with the interface between phases (Clift, 1978). These mechanisms are not well understood so major commercially available CFD codes do not have the capability of particle tracking in multiphase flow. Modeling the detailed flow field of multiphase flow is somewhat impractical; obtaining meaningful particle tracking in multiphase flow is an even more challenging mission.

The erosion problem in multiphase flow is a concern for petroleum industry. Due to the limitations of CFD capabilities in multiphase flow modeling and particle tracking, reasonable simplifications have to be made in order to estimate the erosion in multiphase flow. Vertical upward multiphase flow is of particular interest to this research. Thus the focus of this chapter is to predict the erosion in elbows for vertical upward bubbly flow, annular flow, and slug flow adopting the similar approach of erosion prediction in single-phase flow. Assumptions are made according to the characteristics of each flow pattern. Predictions are examined against data available in literature to determine the reasonability of the assumptions.

For a multiphase flow study, the determination of the flow pattern for a given flow condition is important. Several models have been proposed to predict multiphase flow patterns. Examples include Taitel & Duckler (1976) model, Barnea (1986) model,
and Zhang’s (2003) unified model. Among them, Zhang’s model not only predicts the flow pattern, it provides necessary variables that are vital for this research such as liquid holdup of the liquid film and slug body, length scales of slug unit and slug body for slug flow. Hence, Zhang’s model is applied to predict multiphase flow pattern in this research.

**Erosion Prediction in Bubbly Flow**

Erosion predictions in elbows with air/water flow with the conditions reported by Salama (1998) are made. The flow conditions are summarized in Table IX-1. According to Zhang’s model, dispersed bubbly flow pattern is predicted for these flow conditions, as shown in the flow pattern map (Figure IX-1).

![Flow Pattern Map](image)

(a) D= 49 mm  
(b) D= 26.5 mm

**Figure IX-1. Predicted Dispersed Bubbly Flow on the Flow Pattern Map.**

For bubbly flow, it is assumed that the flow is homogenous. With this assumption, the air/water bubbly flow is simplified as a single-phase flow where the fluid possesses the mixture fluid properties of the two phases. The CFD-based erosion prediction
procedure is adopted to estimate the erosion for bubbly flow by predicting the erosion for this single-phase. The mixture velocity of the two phases (sum of gas and liquid superficial velocities) is assigned as the inlet velocity of this single-phase flow. Table IX-1 also contains the comparison of the predictions with the data presented by Salama (1998). It is noted that information such as pipe wall material is not reported by Salama. In predictions, the properties of Carbon Steel with Brinell hardness of 120 are used. Meanwhile, a semi-rounded sand sharpness factor is applied. The factor, $f_m$, is calculated by dividing the prediction by the data. Experimental results in Chapter VI have confirmed that the CFD-based single-phase erosion model produces an over-prediction factor, $f_s$, which is about 10. In order to eliminate the over-prediction factor induced by the single-phase erosion prediction model in evaluating the proposed erosion prediction approach, an approach factor, $f_a$, is introduced and is defined as $f_a = f_m / f_s$. $f_a$ indicates the deviation of prediction which is caused by the adopted assumption and erosion estimate approach from experimental data. In this research, $f_a$ is approximately determined as $f_m / 10$. $f_a$ rather than $f_m$ is presented in Table IX-1 as well as later discussions.

**Table IX-1. Predicted Erosion for Bubbly Flow in Elbows and the Data Reported by Salama (1998).**

<table>
<thead>
<tr>
<th>Case</th>
<th>Vsg (m/s)</th>
<th>Vsl (m/s)</th>
<th>D (mm)</th>
<th>Data (mm/kg)</th>
<th>Prediction / $f_s$ (mm/kg)</th>
<th>Factor ($f_a$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>5</td>
<td>49</td>
<td>1.35e-5</td>
<td>1.67e-5</td>
<td>1.3</td>
</tr>
<tr>
<td>2</td>
<td>3.5</td>
<td>4</td>
<td>49</td>
<td>4.60e-5</td>
<td>2.91e-5</td>
<td>0.65</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>6.2</td>
<td>26.5</td>
<td>1.80e-4</td>
<td>1.136e-4</td>
<td>0.63</td>
</tr>
</tbody>
</table>
As seen in Table IX-1, the predicted approach factor $f_a$ of erosion in bubbly flow varies in a range, from 0.63 to 1.3, which is deemed reasonable for erosion predictions. Therefore the proposed approximation method for erosion prediction in bubbly flow is effective.

**Erosion Prediction in Annular Flow**

**Annular Flow and Annular-Mist Flow**

Vertical, upward gas/liquid two-phase annular pipe flow is characterized by the continuous thin liquid film on the pipe wall and the continuous gas flow in the core region. In addition, a portion of the liquid is entrained in the gas core flow in the form of liquid droplets formed from the “tip” of the wavy liquid film. This phenomenon is called liquid entrainment. Entrainment ratio, $E$, is defined as the percentage of the mass of liquid that is entrained in the gas core region. Entrained liquid droplets may return to the continuous liquid film, which is so-called droplet deposition. For a fully developed annular flow with no phase change, the entrainment and deposition processes are in equilibrium, which implies that the liquid entrainment ratio stays constant. Annular flow with liquid entrainment and deposition is schematically exhibited in Figure IX-2.

As will be discussed later in this section, the liquid entrainment ratio is determined by the flow parameters among which the Reynolds number of the gas core flow is the key factor. For annular flow with very high gas velocity, the intensive turbulence on the gas/liquid interface promotes liquid droplet entrainment such that the entrainment ratio is close to 1. In this case, the continuous liquid film is unable to form
and discontinuous liquid droplets are observed on the wall. This type of annular flow is classified as annular-mist flow.

![Diagram of Vertical Upward Annular Flow]

**Figure IX-2. Schematic of Vertical Upward Annular Flow.**

**Air/Water/Sand Annular Flow**

When sand particles are entrained in the annular flow, particles are present in the liquid film, gas core region, as well as the entrained liquid droplets, as shown in Figure IX-3. In the gas core region, the flow of gas/droplets/sand can be treated as homogenous flow. The sand particles in the liquid film, the gas core and the liquid droplets contribute to erosion in the elbow. In general, the velocity of the liquid film is much less than the gas core velocity. In addition, the movement of particles in the liquid film is well confined because of the high density and viscosity of water. The liquid film also acts as a barrier for the sand particles in the gas core region to impinge the wall of elbow. The
erosion due to the sand in the liquid film is negligible as compared to that due to the sand in the gas core region (including liquid droplets). Thus the erosion problem of annular flow is simplified as the erosion problem of the homogenous gas core flow that can be solved using a similar approach as single-phase flow.

![Cross-Sectional View of Air/Water/Sand in Annular Flow.](image)

**Figure IX-3. Cross-Sectional View of Air/Water/Sand in Annular Flow.**

The cushion effect of the liquid film is accounted for by introducing an artificial liquid film during the particle tracking process. A User-Subroutine was developed and implemented in CFX to accomplish particle tracking in the artificial liquid film. During the particle tracking in the gas core flow using the single-phase approximation, once the particle impinges the artificial wall, this subroutine is activated so that the particles have to penetrate through the artificial liquid film before they reach the wall. As seen in Figure IX-4, the thickness of the artificial liquid film, $\delta$, is determined by the mechanistic model of Ansari (1994). The particle tracking model used in this subroutine is Equation (III-6)
with the absence of pressure gradient and rotating coordinate forces. A linear velocity distribution is assigned for the liquid film, as seen in Figure IX-5. $V_{\text{film}}$ is the average liquid film velocity calculated by the mechanistic model of Ansari et al. (1994).

![Figure IX-4. Compensation of Liquid Film.](image)

![Figure IX-5. Linear Velocity Distribution in the Artificial Liquid Film.](image)

With the simplifications made above, the determination of the mass of sand in the gas core region is critical to estimate the erosion in the elbow. In some of his experiments Santos (2002) shows that the mass ratio of sand particles in the gas core region is almost the same as the liquid entrainment ratio. Thus the determination of the liquid entrainment ratio helps to quantify the mass ratio of sand particles in the gas core region.
Liquid Entrainment Ratio in Annular Flow

The interfacial structure between the gas core and liquid film is highly complex; a variety of mechanisms of droplet entrainment is described in the literature (Hewitt, 1970 and Ishii, 1975). However, in most flow conditions of interest, the phenomenon dominating the droplet formation process at the phase interface is considered to be the shearing-off of roll wave crests by the turbulent gas flow: the tips of roll waves are drawn out by the interfacial shear force against the retaining force of surface tension, then the portions of waves are broken up into a number of small atomized droplets (Okawa et al., 2002).

Extensive effort has been made to develop the correlations of liquid entrainment. Representative work has been done by Wallis (1965), Asali et al. (1985), Govan et al. (1988), Ishii and Mishima (1989), Sugawara (1990), and Okawa et al. (2002). The correlation proposed by Ishii is based on experimental data sets of water and shows good accuracy for air-water flow conditions. The liquid droplet entrainment predictive correlation proposed by Ishii and Mishima (1989), given in Equation IX-1, hereon is used to calculate the entrainment ratio in this research.

\[
E = \tanh(7.25 \times 10^{-7} \text{We}^{1.25} \text{Re}_{l}^{0.25}) \tag{IX-1}
\]

where \( \text{We} \) is the Weber number and is determined by

\[
\text{We} = \frac{\rho_g V_{sg}^2 D}{\sigma} \left( \frac{\rho_1 - \rho_g}{\rho_g} \right)^{\frac{1}{3}} \tag{IX-2}
\]

and where \( \text{Re}_{l} \) is the total liquid Reynolds number and is defined as

\[
\text{Re}_{l} = \frac{\rho_1 V_{sl} D}{\mu_1} \tag{IX-3}
\]
where \( V_{sg} \) and \( V_{sl} \) are the superficial velocity of air and water, respectively; \( \sigma \) is the surface tension coefficient of water; and \( D \) is the pipe diameter.

**Erosion Prediction for Elbows in Annular Flow**

Summarizing the discussion of the previous two sub-sections, the approach employed in this research to predict the erosion in elbows for air/water/sand annular flow is based on the following treatments:

1) Sand erosion is the summation of the contribution of sand in two regions: liquid film and gas core region;

2) However, the dominant erosion is caused by the sand in the gas core region and the erosion due to the sand in the liquid film is considered negligible;

3) It is assumed that the sand mass ratio in the gas core region is equal to the liquid droplet entrainment ratio which is calculated by Equation (IX-1);

4) Air, droplets, and sand particles flowing in the gas core region are treated as a homogenous flow;

5) Erosion is predicted for a single-phase flow using mixture properties of the gas core region;

6) A User-Subroutine is called when the particle strikes the wall (artificial) to account for the liquid film which provides the cushioning effect of the film;

7) The erosion is determined by applying the erosion obtained for the single-phase flow with treatment multiplied by the liquid droplet entrainment ratio.

Erosion was predicted for flow conditions reported by Salama (1998) as listed in Table IX-2 applying the proposed erosion estimate method. The predicted flow pattern for these conditions is shown in Figure IX-6. Table IX-2 also contains the predicted
results and the data reported by Salama (1998). Simulations were conducted to identify the significance of the artificial liquid film treatment. Predicted erosion is compared with data in Table IX-3 without applying the artificial liquid film subroutine.

![Flow Pattern Map](image)

(a) D= 49 mm  
(b) D= 26.5 mm

Figure IX-6. Predicted Annular Flow on the Flow Pattern Map.

<table>
<thead>
<tr>
<th>Case</th>
<th>Vsg (m/s)</th>
<th>Vsl (m/s)</th>
<th>D (mm)</th>
<th>d (µm)</th>
<th>Data (mm/kg)</th>
<th>Prediction / ( f_s ) (mm/kg)</th>
<th>Factor (( f_a ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>30</td>
<td>1</td>
<td>49</td>
<td>150</td>
<td>5.52e-4</td>
<td>6.24e-4</td>
<td>1.13</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>0.5</td>
<td>49</td>
<td>150</td>
<td>2.46e-3</td>
<td>1.05e-3</td>
<td>0.49</td>
</tr>
<tr>
<td>6</td>
<td>35</td>
<td>1</td>
<td>26.5</td>
<td>250</td>
<td>6.56e-3</td>
<td>3.66E-03</td>
<td>0.55</td>
</tr>
<tr>
<td>7</td>
<td>34.3</td>
<td>0.5</td>
<td>26.5</td>
<td>250</td>
<td>7.20e-3</td>
<td>4.22E-03</td>
<td>0.59</td>
</tr>
<tr>
<td>8</td>
<td>37</td>
<td>0.7</td>
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<td>250</td>
<td>8.03e-3</td>
<td>5.14E-03</td>
<td>0.64</td>
</tr>
<tr>
<td>9</td>
<td>38.5</td>
<td>0.5</td>
<td>26.5</td>
<td>250</td>
<td>8.03e-3</td>
<td>5.73E-03</td>
<td>0.71</td>
</tr>
<tr>
<td>10</td>
<td>44</td>
<td>1.5</td>
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<td>250</td>
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<td>1.07E-02</td>
<td>1.01</td>
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<tr>
<td>11</td>
<td>51</td>
<td>0.6</td>
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<td>250</td>
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<td>1.59E-02</td>
<td>1.19</td>
</tr>
<tr>
<td>12</td>
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<td>0.7</td>
<td>26.5</td>
<td>250</td>
<td>1.33e-2</td>
<td>1.69E-02</td>
<td>1.27</td>
</tr>
</tbody>
</table>

Table IX-2. Predicted Erosion (with Artificial Liquid Film Treatment) in the Elbow for Annular Flow and the Data Reported by Salama (1998).
Table IX-3. Predicted Erosion (without Artificial Liquid Film Treatment) in the Elbow for Annular Flow with and the Data Reported by Salama (1998).

<table>
<thead>
<tr>
<th>Case</th>
<th>Vsg (m/s)</th>
<th>Vsl (m/s)</th>
<th>D (mm)</th>
<th>d (µm)</th>
<th>Data (mm/kg)</th>
<th>Prediction / ( f_a ) (mm/kg)</th>
<th>Factor (( f_a ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>30</td>
<td>1</td>
<td>49</td>
<td>150</td>
<td>5.52e-4</td>
<td>1.22e-3</td>
<td>2.21</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>0.5</td>
<td>49</td>
<td>150</td>
<td>2.46e-3</td>
<td>1.38e-3</td>
<td>0.56</td>
</tr>
<tr>
<td>6</td>
<td>35</td>
<td>1</td>
<td>26.5</td>
<td>250</td>
<td>6.56e-3</td>
<td>4.66e-3</td>
<td>0.71</td>
</tr>
<tr>
<td>7</td>
<td>34.3</td>
<td>0.5</td>
<td>26.5</td>
<td>250</td>
<td>7.20e-3</td>
<td>5.11e-3</td>
<td>0.71</td>
</tr>
<tr>
<td>8</td>
<td>37</td>
<td>0.7</td>
<td>26.5</td>
<td>250</td>
<td>8.03e-3</td>
<td>5.62e-3</td>
<td>0.70</td>
</tr>
<tr>
<td>9</td>
<td>38.5</td>
<td>0.5</td>
<td>26.5</td>
<td>250</td>
<td>8.03e-3</td>
<td>7.87e-3</td>
<td>0.98</td>
</tr>
<tr>
<td>10</td>
<td>44</td>
<td>1.5</td>
<td>26.5</td>
<td>250</td>
<td>1.05e-2</td>
<td>1.10e-2</td>
<td>1.05</td>
</tr>
<tr>
<td>11</td>
<td>51</td>
<td>0.6</td>
<td>26.5</td>
<td>250</td>
<td>1.34e-2</td>
<td>1.84e-2</td>
<td>1.37</td>
</tr>
<tr>
<td>12</td>
<td>52</td>
<td>0.7</td>
<td>26.5</td>
<td>250</td>
<td>1.33e-2</td>
<td>1.97e-2</td>
<td>1.48</td>
</tr>
</tbody>
</table>

Table IX-2 indicates that the approach factor \( f_a \) lies in the fair range of 0.49 ~ 1.27. Results obtained by not applying the artificial liquid film are seen in Table IX-3, \( f_a \) varies within the range of 0.56~2.21. Comparison of Tables IX-2 and IX-3 shows that including the retarding effect of the liquid film can significantly affect predicted erosion and that the approach factor \( f_a \) is greatly narrowed when artificial liquid film is applied. In general, the proposed erosion estimate method (applying the liquid film) gives a reasonable estimate of erosion for annular flow.

The proposed erosion estimate approach was applied to flow conditions reported by Bourgooyne (1989). Table IX-4 lists the flow conditions and summarizes the data as well as predictions. The predicted flow patterns are shown in Figure IX-7. For all conditions listed in Table IX-5 the liquid droplet entrainment ratio calculated by Equation...
IX-2 is unity, which implies that the liquid phase only exists as liquid droplets in the gas core region and thus annular-mist flow is formed. The liquid film is not applicable for annular-mist flow.

![Flow Pattern Map](image)

**Figure IX-7. Predicted Annular-Mist Flow on the Flow Pattern Map.**

<table>
<thead>
<tr>
<th>Case</th>
<th>Vsg (m/s)</th>
<th>Vsl (m/s)</th>
<th>D (mm)</th>
<th>d (µm)</th>
<th>Data (mm/kg)</th>
<th>Prediction / $f_s$ (mm/kg)</th>
<th>Factor ($f_s$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>86</td>
<td>0.53</td>
<td>52.5</td>
<td>250</td>
<td>1.27e-1</td>
<td>1.93e-2</td>
<td>0.15</td>
</tr>
<tr>
<td>14</td>
<td>92</td>
<td>0.53</td>
<td>52.5</td>
<td>250</td>
<td>1.21e-1</td>
<td>2.21e-2</td>
<td>0.18</td>
</tr>
<tr>
<td>15</td>
<td>89</td>
<td>0.12</td>
<td>52.5</td>
<td>250</td>
<td>1.08e-1</td>
<td>2.27e-2</td>
<td>0.21</td>
</tr>
<tr>
<td>16</td>
<td>84</td>
<td>0.53</td>
<td>52.5</td>
<td>250</td>
<td>9.34e-2</td>
<td>1.91e-2</td>
<td>0.20</td>
</tr>
<tr>
<td>17</td>
<td>72</td>
<td>0.53</td>
<td>52.5</td>
<td>250</td>
<td>5.37e-2</td>
<td>1.45e-2</td>
<td>0.27</td>
</tr>
<tr>
<td>18</td>
<td>84</td>
<td>0.12</td>
<td>52.5</td>
<td>250</td>
<td>7.51e-2</td>
<td>1.99e-2</td>
<td>0.26</td>
</tr>
<tr>
<td>19</td>
<td>92</td>
<td>0.12</td>
<td>52.5</td>
<td>250</td>
<td>9.94e-2</td>
<td>2.35e-2</td>
<td>0.23</td>
</tr>
<tr>
<td>20</td>
<td>107</td>
<td>0.53</td>
<td>52.5</td>
<td>250</td>
<td>1.05e-1</td>
<td>2.87e-2</td>
<td>0.28</td>
</tr>
</tbody>
</table>

**Table IX-4. Predicted Erosion in the Elbow for Annular-Mist Flow and the Data Reported by Bourgoyne (1989).**
From Table IX-4, the approach factor \( f_a \) fluctuates between 0.15 and 0.28. It is clear that the proposed model constantly under-predicts the erosion of annular-mist flow. Several reasons may contribute to this under-prediction. First, very high sand rates were used by Bourgoyn (1989). A significant increase of target (elbow) temperature was observed, which could result in a considerable change of material properties and thus promote the erosion process that is not captured by the current erosion prediction model. Secondly, Ahlert (1994) has reported the “wet” erosion phenomenon: if a little amount of liquid that is just sufficient to wet the target surface was injected, the erosion on the wet target was found to be about twice as the dry target. This finding implies that the erosion mechanism can be significantly influenced by the presence of low liquid injection. For high air velocity and very low liquid loading flows listed in Table IX-4, the “wet” erosion phenomenon may occur. Despite of these uncertainties, fair agreement between the predictions and the data is achieved with the simplified erosion prediction approach. This study also suggests that further investigation of erosion for very high gas velocity and low liquid injection is much needed.

**Erosion Prediction in Slug Flow**

**Air/Water Slug Flow**

Slug flow is an extremely complex flow pattern. Figure IX-8 shows the schematic of a vertical upward slug flow. Three distinct flow phenomena co-exist in a slug unit: bullet-shaped Taylor bubble, continuous liquid film between the Taylor bubble and the wall, and a slug body where the liquid and air bubbles form a dispersed bubbly flow. Slug flow is essentially a transient flow: the front of the Taylor bubble is moving at the
translational velocity, \( V_{TB} \); the gas packet (Taylor bubble) has the velocity of \( V_{GTB} \); the velocity of the slug body is \( V_s \); while the liquid film velocity is \( V_{LTB} \) which decreases from its maximum value at the back of the slug body to the minimum value at the front of the slug body. In general, the velocity distribution follows \( V_{TB} > V_{GTB} > V_s > V_{LTB} \). In the analysis of slug flow a homogenous no slip flow is usually assumed in the slug body, namely, \( V_{GLS} = V_{LLS} = V_s \) where \( V_{GLS} \) and \( V_{LLS} \) are the superficial velocity of the gas and liquid phases, respectively. It also happens that the liquid slides downward in the film aside the uprising Taylor bubbles.

\[
\text{Figure IX-8. Schematic of Vertical Upward Slug Flow.}
\]

**Erosion Prediction in Slug Flow**

In air/water/sand slug flow, sand particles are entrained in the slug body as well as the liquid film. The possibility also exists for particles to be in the gas core (Taylor
bubble). The gas packet is rising faster than the slug body. This slippage will allow the slug body to pick up the particles in the gas packet in which the inertia force is dominating the motion of particles. Therefore in this study an assumption is made that no particles are in the gas packet for fully developed slug flow.

Since sand erosion in slug flow is such a complex problem, two erosion estimate approaches were developed in this research. Simulations were performed applying each of these approaches to predict the erosion for the flow conditions reported by Salama (1998). A comparison was conducted to identify the appropriate erosion prediction approach that is suitable for slug flow.

Similar to the analysis and simplifications for the erosion estimation in annular flow, the first erosion prediction procedure was proposed for slug flow based on the following assumptions and treatments:

1) Flow in the slug body is homogenous;
2) No sand particles are entrained in the gas packet for fully developed slug flow;
3) Erosion is caused by the sand in the slug body as well as the liquid film;
4) Sand distribution in the slug body and the liquid film is the same as the liquid mass ratio in these two regions;
5) The liquid film velocity is generally considerably low as compared to the slug body or even negative (for vertical upward slug flow). The erosion due to the sand in liquid film is negligible;
6) Erosion is predicted for the single-phase flow using the homogenous mixture fluid properties of the slug body. Simulated erosion results are multiplied by the sand mass ratio to yield the predicted erosion for the slug flow.
This approach was applied to flow conditions presented in Table IX-5. Flow patterns predicted by Zhang’s model (2003) are shown in Figure IX-9. In this approach, Zhang’s model is also used to calculate necessary slug flow parameters including slug body and liquid film length, liquid holdup in the slug as well as liquid film, and film velocity.

![Figure IX-9. Predicted Slug Flow on the Flow Pattern Map.](image)

<table>
<thead>
<tr>
<th>Case</th>
<th>Vsg (m/s)</th>
<th>Vsl (m/s)</th>
<th>D (mm)</th>
<th>d (µm)</th>
<th>Data (mm/kg)</th>
<th>Prediction / $f_s$ (mm/kg)</th>
<th>Factor ($f_a$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>20</td>
<td>3.1</td>
<td>49</td>
<td>150</td>
<td>6.93e-5</td>
<td>5.96e-5</td>
<td>0.86</td>
</tr>
<tr>
<td>22</td>
<td>15</td>
<td>1</td>
<td>45</td>
<td>150</td>
<td>1.47e-4</td>
<td>6.87e-6</td>
<td>0.046</td>
</tr>
<tr>
<td>23</td>
<td>10</td>
<td>0.7</td>
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<td>150</td>
<td>7.01e-5</td>
<td>3.19e-6</td>
<td>0.045</td>
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<tr>
<td>24</td>
<td>8</td>
<td>0.2</td>
<td>49</td>
<td>150</td>
<td>1.23e-4</td>
<td>2.17e-6</td>
<td>0.018</td>
</tr>
<tr>
<td>25</td>
<td>14.4</td>
<td>1.5</td>
<td>26.5</td>
<td>250</td>
<td>2.30e-4</td>
<td>1.68e-4</td>
<td>0.73</td>
</tr>
<tr>
<td>26</td>
<td>14.6</td>
<td>1.5</td>
<td>26.5</td>
<td>250</td>
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<td>0.39</td>
</tr>
<tr>
<td>27</td>
<td>34</td>
<td>2.1</td>
<td>26.5</td>
<td>250</td>
<td>2.83e-3</td>
<td>7.24e-4</td>
<td>0.26</td>
</tr>
</tbody>
</table>
As seen in Tables IX-6, it is obvious that the erosion is under-predicted for slug flow. Even the order of magnitude of the factor between the prediction and the data varies. The analysis and simplifications made in the first approach are logical and physical in straight pipe, but the erosion occurs at the elbow bend section. Poor agreement observed in Table IX-5 between the predictions and the data indicates that the first approach does not reflect the flow characteristics in the elbow section. According to Zhang’s (2003) model, the liquid holdup of the slug body is about 0.36 for the cases of interest, which determines that the slug body has almost constant mixture density and viscosity (0.361 kg/m$^3$ and 0.00037 Pa·s). Simulations show that the predicted erosion for these mixture properties is very low, which results in under-predicted erosion. Thus lower mixture fluid properties (density and viscosity) than the slug body are desired for simulations. In fact, mixing of the slug unit is expected in the elbow bend. It can be deduced that the fluid in the elbow where the erosion takes place has lower fluid mixture properties than in the slug body.

Based on the first approach and the previous discussion, the second approach for slug flow is summarized as follows:

1) The slug unit is homogenous in the elbow bend section;
2) Sand particles are only entrained in the slug body and liquid film;
3) Sand mass ratio in the slug body and liquid film is the same as the liquid mass ratio in these two regions;
4) Erosion due to sand particles in the liquid film is neglected;
5) Erosion is predicted for a single-phase flow that possesses mixture fluid properties of the slug flow. This result is weighted by the sand mass ratio in the slug body to give the final predicted erosion.

For identical flow conditions shown in Tables IX-5 the second erosion approximation method is applied. Table IX-6 lists the simulation results and the comparison with the data.


<table>
<thead>
<tr>
<th>Case</th>
<th>Vsg (m/s)</th>
<th>Vsl (m/s)</th>
<th>D (mm)</th>
<th>d (µm)</th>
<th>Data (mm/kg)</th>
<th>Prediction / $f_s$ (mm/kg)</th>
<th>Factor ($f_s$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>20</td>
<td>3.1</td>
<td>49</td>
<td>150</td>
<td>6.93e-5</td>
<td>2.50e-4</td>
<td>3.61</td>
</tr>
<tr>
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<td>150</td>
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<td>8.59e-5</td>
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<tr>
<td>23</td>
<td>10</td>
<td>0.7</td>
<td>49</td>
<td>150</td>
<td>7.01e-5</td>
<td>3.78e-5</td>
<td>0.54</td>
</tr>
<tr>
<td>24</td>
<td>8</td>
<td>0.2</td>
<td>49</td>
<td>150</td>
<td>1.23e-4</td>
<td>6.69e-5</td>
<td>0.55</td>
</tr>
<tr>
<td>25</td>
<td>14.4</td>
<td>1.5</td>
<td>26.5</td>
<td>250</td>
<td>2.30e-4</td>
<td>5.99e-4</td>
<td>2.60</td>
</tr>
<tr>
<td>26</td>
<td>14.6</td>
<td>1.5</td>
<td>26.5</td>
<td>250</td>
<td>4.20e-4</td>
<td>5.88e-4</td>
<td>1.40</td>
</tr>
<tr>
<td>27</td>
<td>34</td>
<td>2.1</td>
<td>26.5</td>
<td>250</td>
<td>2.83e-3</td>
<td>3.48e-3</td>
<td>1.23</td>
</tr>
</tbody>
</table>

As compared to the range 0.018 – 0.86 of the approach factor $f_a$ provided by the first approach, Table IX-6 shows that the value of $f_a$ predicted by the second approach has a more fair range, 0.54 –3.61. The second approach contains more physics of the slug flow in the elbow bend, so the erosion is more accurately predicted.
Summary

Prediction of sand erosion in elbows for vertical upward air/water bubbly flow, annular flow, annular-mist flow and slug flow is the focus of this chapter. Because of the immaturity of CFD in multiphase flow modeling and particle tracking, appropriate assumptions and simplifications have to be made according to the characteristics of individual multiphase flow patterns in order to tackle sand erosion problems using the established CFD-based single-phase erosion prediction procedure. This idea leads to the development of erosion estimate approaches for multiphase flow that reflect key flow characteristics of the flow pattern and also includes the application of the single-phase CFD-based erosion prediction procedure. Accordingly, erosion estimate approaches were proposed for bubbly flow, annular flow (and annular-mist), and slug flow, separately:

a) Erosion in a bubbly flow is converted to an erosion problem in a single-phase flow that possesses fluid mixture properties.

b) Erosion in annular flow is simplified as the erosion of the homogenous gas core flow with the compensation of artificial liquid film. Accurate determination of sand mass ratio (equal to the liquid droplet entrainment ratio) in the gas core is important.

c) Erosion in slug flow is assumed to be only due to the sand in the slug body. Two types of fluid properties were tested: homogenous slug body and homogenous slug unit. Simulation results confirm that more reasonable erosion is predicted applying the mixture fluid properties of the slug unit (or slug flow).
Erosion predictions were conducted applying these approximate methods for the multiphase flow conditions presented by Bourgoyne (1989) and Salama (1998). It must be noted that the multiphase erosion prediction approaches inherit the over-prediction feature of the CFD-based single-phase erosion prediction procedure. Hence, exclusion of the single-phase erosion model over-prediction factor, $f_s$, is required to accurately access the applicability of the proposed multiphase erosion prediction approaches. Consequently, in order to be comparable with experimental data, the predicted erosion was divided by $f_s$ in this study. As to facilitate further discussion, predicted erosion as well as experimental data for bubbly flow, annular and annular-mist flow, and slug flow in Tables IX-1, IX-2, and IX-4, and IX-6 are re-assembled in Figure IX-10.

Figure IX-10. Comparison of the Predicted Erosion in Elbows for Multiphase Flow with Experimental Data Reported by Bourgoyne (1989) and Salama (1998).
Good agreement between the simulations and the data of bubbly flow is exhibited in Figure IX-10. Although scattered, predictions for slug flow deviate from the experimental data within a fair range. Predictions also show reasonable agreement with the experimental data for annular flow. For annular-mist flow the erosion is under-predicted by a fair factor and repeated under-prediction of erosion is observed. Discussions in the related section have pointed out the possibilities that may cause the under-prediction. From Figure IX-10 one can expect that reasonable estimates of erosion can be achieved by applying the proposed erosion approximation approaches for bubbly flow, annular flow, and slug flow. For annular-mist flow, a correction factor of erosion may need to be applied or more reliable data may be required.

In spite of the extreme complexity of multiphase flow and the diversity of parameters that govern the erosion process, the attempts of this study provide an effective tool to estimate erosion in multiphase flow before more thorough flow modeling and particle tracking in multiphase flow become practical.

This research can be deemed as an original and preliminary attempt to combine the mechanistic and CFD approaches to predict the erosion in multiphase flows. Proposed erosion estimate methods were evaluated by literature data. Effort is required to further validate these approaches.
CHAPTER X
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary
There are two primary goals of this study. The first one is to determine the relative erosion resistance between plug tees and elbows for single-phase flow. The other aspect is to shed some light on the erosion in elbows for multiphase flow using a relatively simplistic approach. Implementations and results will be discussed for each branch.

For single-phase flow, a CFD-based erosion prediction procedure was applied to predict the erosion in elbows and plug tees for a broad range of flow conditions. This CFD-based erosion prediction procedure consists of three steps: flow simulation, particle tracking, and erosion calculation. This CFD-based erosion prediction procedure is significantly improved by implementing a stochastic particle rebound model (Grant and Tabakoff, 1975). The influence of particle rebound models (Forder’s non-stochastic model, 1998, and Grant and Tabakoff’s stochastic model, 1975) on particle tracking and erosion patterns in the elbows and plug tees is investigated. The penetration ratio between the plug tee (both End Region and Side & Corner Region) and the elbow is determined numerically by applying the implemented stochastic particle rebound model.

Simulations illustrate that plug tees experience greater erosion than elbows for conditions where the carrier fluid has high density and viscosity, like water. Predictions
agree with Bourgyne’s (1989) statement based on experimental observations: “when only liquid and sand are present, the plug tee is less erosion resistant than the long radius or short radius elbow.”

Experimental erosion tests for low sand volume concentration (0.00025% ~ 0.001%) in the elbow and plug tee with 1-inch diameter were performed to validate predictions obtained in air. According to simulations, particle recirculation in the plug tee section is the dominant factor that determines the erosion pattern as well as the quantity of erosion in the plug tee, and the intensity of particle recirculation depends on particle diameter. Comparison between predicted erosion and measured erosion indicates that particle recirculation is over-predicted for small particle sizes. Based on experimental data and simulation results, it is recommended that the penetration ratio between the End Region of the plug tee and the elbow is 0.5. By distinguishing the contribution of erosion by the first impinge of a particle and impingements resulting from particle recirculation, the empirical particle recirculation correction factor is introduced. Based on simulation results applying the determined particle recirculation correction factor, a correlation is developed to calculate the penetration ratio between the Side & Corner Region of the plug tee to the elbow for air.

Further experimental erosion tests were carried out for high sand volume concentration (0.016% ~ 0.0244%) to preliminarily examine the effect of sand volume concentration on the relative erosion resistance between the plug tee and the elbow in air. Three significant differences are observed from these tests as compared to low sand volume concentration tests: negligible erosion is detected on the plug tee end surface;
high erosion occurs at the plug tee exit; the erosion of plug tee is less than the elbow by about one to two-orders of magnitude.

As for multiphase flow, a preliminary effort was made to model the bubbly flow in a vertical upward channel with blockage (Tapucu, 1983) and to simulate the uprising air Taylor bubble from stagnant water in a tube using the multi-fluid model in CFX. But tremendous model improvements and numerical implementation are required to enable the currently available multi-fluid model to successfully simulate slug flows and annular flows. Immaturity of multiphase flow modeling and particle tracking determines that the single-phase CFD-based erosion prediction procedure can not be simply adopted for erosion prediction in multiphase flow.

A combination of a mechanistic and CFD approach is proposed to estimate the erosion in elbows for multiphase flow. The key of this approach is to identify a representative single-phase flow and to determine the effective sand mass ratio that result in an accurate erosion prediction for individual flow patterns. The single-phase CFD based erosion prediction procedure is employed to calculate the erosion for the resulting representative single-phase flow. This result is weighted by the effective sand mass ratio to yield the estimated erosion in multiphase flow. The flow regimes covered in this study include bubbly flow, slug flow, and annular/annular-mist flow. The details of the treatments for individual flow regimes are listed as follows:

**Bubbly Flow**  
The representative single-phase flow has the mixture properties of two phases of the bubbly flow. The effective mass ratio is unity.
 Slug Flow  The representative single-phase flow has the mixture properties of two phases of the slug flow (or slug unit). The effective mass ratio is the mass ratio of liquid in the slug body.

Annular/annular-mist Flow  The representative single-phase flow has the mixture properties of two phases of the gas core flow. The effective sand mass ratio is the equal to liquid droplet entrainment ratio (which is unity for annular-mist flow). In addition, the liquid film effect on particle tracking and erosion is accounted for by introducing a liquid film sub-routine when a particle strikes the wall.

The proposed erosion estimate approaches are validated by experimental data by Bourgoyne (1989) and Salama (1998). Reasonable agreement is achieved between the predictions and data for bubbly flow, slug flow, and annular flow. For annular-mist flow, the erosion is consistently under-predicted. Several reasons that may contribute to the under-prediction are identified.

Conclusions

Conclusions drawn from studies of erosion in single-phase flow:

1) In water, simulation results show that the erosion in the plug tee is greater than the elbow. This agrees with Bourgoyne’s experimental observation.

2) In order to realistically predict the erosion, the utilization of the stochastic particle rebound model is critical for the case where the carrier fluid has low density and viscosity (such as air), and strong particle recirculation is anticipated, or the erosion caused by localized particle impingements other than the first impingement is a concern.
3) In low density and viscosity fluids, the penetration ratio between the plug tee and the elbow is dominantly controlled by sand volume concentration. Arbitrarily applying the results obtained from a given sand volume concentration is dangerous.

4) In low density and viscosity fluids, the penetration ratio between the End Region of the plug tee and the elbow is recommended as 0.5.

5) In low density and viscosity fluids, the penetration ratio between the Side & Corner Region of the plug tee and the elbow is mainly affected by particle diameter and pipe diameter.

Conclusions drawn from studies of erosion in multiphase flow:

1) Currently, thorough flow field simulation and particle tracking is an impractical approach to predict the erosion for multiphase flow.

2) The single-phase CFD-based erosion prediction procedure can be applied to estimate the erosion in multiphase flow provided that appropriate assumptions and necessary simplifications are made based on the analysis of the characteristic flow behavior of individual flow regimes.

**Recommendations**

Based on findings of the present study, recommendations are suggested in both experimental and numerical aspects.

Experimentally:
1) The predicted erosion in the plug tee and elbow in water should be validated by experimental data, although it is a fact that the general trend of the prediction agrees with Bourgyne’s observation.

2) The profilometer is suitable to determine the local thickness loss on the erosion test specimen one-dimensionally and can be used to measure the maximum thickness loss of the specimen surface two-dimensionally. The confidence on the experimental data would increase if the thickness loss profile had been measured three-dimensionally.

3) More accurate flowmeters are desired to closely monitor the actual flow conditions in the test flow loop.

4) Efforts should be made to further examine the influence of sand volume concentration on the effectiveness of applying a plug tee instead of an elbow.

5) The proposed erosion estimate approaches should be validated by experimental data in a broader sense.

Numerically:

1) Numerical efforts should be made to predict the penetration ratio for compressed gases.

2) The developed mechanistic model that calculates the penetration ratio between plug tees and elbows should account for the impact of fluid properties. This can be possible only if the above recommendation is accomplished.

3) Although significant improvements are obtained by applying the stochastic particle rebound model, particle recirculation in the plug tee is over-predicted for small particle sizes. The reasons behind this numerical discrepancy should be
investigated. It is suggested that the further study can focus on the following areas:

- Flow field solution in the plug tee
- Turbulence parameters (eddy length and eddy life)
- Particle tracking model
- Fluid-particle interaction (using two-way coupling particle tracking method)
- Particle-particle interaction

4) As deduced from experimental data, particle-particle interaction plays a key role in determining the erosion in high sand volume concentration. Model improvements and numerical implementations are desired to account for this particle-particle interaction effect so that the CFD model is able to reproduce the experimental flow conditions.
### NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Pipe wall material dependent empirical constant</td>
</tr>
<tr>
<td>A&lt;sub&gt;j&lt;/sub&gt;</td>
<td>Area of the local impinged computational cell face (m&lt;sup&gt;2&lt;/sup&gt;)</td>
</tr>
<tr>
<td>B</td>
<td>Body force (N)</td>
</tr>
<tr>
<td>C</td>
<td>Constant recommended by API RP 14E</td>
</tr>
<tr>
<td>C&lt;sub&gt;D&lt;/sub&gt;</td>
<td>Drag coefficient</td>
</tr>
<tr>
<td>C&lt;sub&gt;L&lt;/sub&gt;</td>
<td>Lift force coefficient</td>
</tr>
<tr>
<td>C&lt;sub&gt;TD&lt;/sub&gt;</td>
<td>Turbulent dispersion coefficient</td>
</tr>
<tr>
<td>C&lt;sub&gt;VM&lt;/sub&gt;</td>
<td>Virtual mass force coefficient (particle shape dependent constant)</td>
</tr>
<tr>
<td>c&lt;sub&gt;αβ&lt;/sub&gt;</td>
<td>Inter-phase transfer coefficient of Φ between phases α and β</td>
</tr>
<tr>
<td>c&lt;sub&gt;αβ&lt;sup&gt;(d)&lt;/sup&gt;&lt;/sub&gt;</td>
<td>Inter-phase drag term</td>
</tr>
<tr>
<td>D</td>
<td>Pipe diameter of the elbow and plug tee</td>
</tr>
<tr>
<td>D&lt;sub&gt;αβ&lt;/sub&gt;</td>
<td>Total drag force per unit volume exerted on particles (N)</td>
</tr>
<tr>
<td>d&lt;sub&gt;p&lt;/sub&gt;</td>
<td>Particle diameter (µm)</td>
</tr>
<tr>
<td>E</td>
<td>Liquid droplet entrainment ratio</td>
</tr>
<tr>
<td>ER</td>
<td>Erosion ratio</td>
</tr>
<tr>
<td>ER&lt;sub&gt;i,j&lt;/sub&gt;</td>
<td>Local erosion ratio due to individual particle impingement</td>
</tr>
</tbody>
</table>
\( e_{\text{par}} \) \hspace{1cm} \text{Particle parallel restitution coefficient}

\( e_{\text{per}} \) \hspace{1cm} \text{Particle perpendicular restitution coefficient}

\( F_c \) \hspace{1cm} \text{Sum of external or body forces (N)}

\( F_s \) \hspace{1cm} \text{Particle shape coefficient}

\( F_A \) \hspace{1cm} \text{Added mass force (N)}

\( F_B \) \hspace{1cm} \text{Buoyancy force (N)}

\( F_D \) \hspace{1cm} \text{Drag force (N)}

\( F_p \) \hspace{1cm} \text{Pressure gradient force (N)}

\( F_R \) \hspace{1cm} \text{Rotating coordinates term to account for centrifugal and Coriolis effects (N)}

\( F_a \) \hspace{1cm} \text{Interfacial non-drag terms (N)}

\( F_\theta \) \hspace{1cm} \text{Particle impact angle dependent function}

\( f_a \) \hspace{1cm} \text{Multiphase erosion prediction method approach factor}

\( f_m \) \hspace{1cm} \text{Multiphase erosion over-prediction factor}

\( f_s \) \hspace{1cm} \text{Single-phase erosion over-prediction factor}

\( I \) \hspace{1cm} \text{Identity matrix}

\( J \) \hspace{1cm} \text{Material properties dependent coefficient}

\( K \) \hspace{1cm} \text{Material properties dependent coefficient}

\( k \) \hspace{1cm} \text{Turbulence kinetic energy}

\( L \) \hspace{1cm} \text{Plug section length of the plug tee}

\( L_e \) \hspace{1cm} \text{Length scale of the randomly sampled turbulence eddy (m)}
\( L_0 \)  Distance for a particle to reach its terminal velocity in air (m)

\( L_T \)  Length scale of the local turbulence dissipation (N)

\( M_p \)  Number of phases

\( \text{MPR} \)  Maximum penetration ratio between the plug tee Side & Corner Region and the elbow

\( \dot{m}_{\alpha\beta} \)  Mass flow rate per unit volume into phase \( \alpha \) from \( \beta \)

\( N_p \)  Total number of particles introduced in the flow

\( \dot{N}_p \)  Particle rate in particles per second flowing

\( n \)  Velocity exponent constant (m/s)

\( p \)  Local fluid pressure (Pa)

\( q \)  Standard deviation

\( \text{Re}_l \)  Total liquid Reynolds number

\( \text{Re}_p \)  Particle Reynolds number

\( \text{Re}_p^* \)  Critical particle Reynolds number

\( \text{Re}_s \)  Particle slip Reynolds number

\( r \)  Turning curve radius of the elbow

\( r_u \)  Volume fractions of multiphase

\( S_u \)  Source term

\( T_e \)  Turbulence eddy lifetime (s)

\( T_i \)  Particle and turbulence eddy interaction time (s)

\( T_k \)  Time for a particle cross the turbulence eddy (s)
\begin{align*}
  t & \quad \text{Time (s)} \\
  U & \quad \text{Instantaneous velocity (m/s)} \\
  u & \quad \text{Mean velocity (m/s)} \\
  \overline{u} & \quad \text{Turbulence flow fluctuating velocity component (m/s)} \\
  V & \quad \text{Magnitude of the particle impact velocity (m/s)} \\
  V_{\text{sg}} & \quad \text{Superficial velocity of gas phase (m/s)} \\
  V_{\text{sl}} & \quad \text{Superficial velocity of liquid phase (m/s)} \\
  V_{1N} & \quad \text{Particle impact velocity normal component (m/s)} \\
  V_{2N} & \quad \text{Particle reflected velocity normal component (m/s)} \\
  V_{1T} & \quad \text{Particle impact velocity tangential component (m/s)} \\
  V_{2T} & \quad \text{Particle reflected velocity tangential component (m/s)} \\
  V_c & \quad \text{Critical particle impingement speed (m/s)} \\
  \overline{V} & \quad \text{Erosional velocity limit (ft/s)} \\
  V_{\text{film}} & \quad \text{Annular flow liquid film average velocity (m/s)} \\
  V_p^{(c)} & \quad \text{Particle’s velocity vector in computational space (m/s)} \\
  V_p & \quad \text{Particle velocity vector (m/s)} \\
  v' & \quad \text{Turbulence flow fluctuating velocity component (m/s)} \\
  \text{We} & \quad \text{Weber number} \\
  w' & \quad \text{Turbulence flow fluctuating velocity component (m/s)} \\
  x_p^{(c)} & \quad \text{Particle’s position vector in computational space (m)} \\
  y_m & \quad \text{Distance to the wall (m)}
\end{align*}
\( \alpha \) One phase in the multiphase flow system

\( \beta \) One phase in the multiphase flow system

\( \Gamma \) Squeeze film correction factor

\( \Gamma_u \) Diffusion coefficient

\( \delta \) Annular flow liquid film thickness (m)

\( \epsilon \) Turbulence dissipation rate (m\(^2\)/s\(^3\))

\( \theta \) Particle impact angle (\(^\circ\))

\( \mu \) Fluid dynamic viscosity (kg/ms)

\( \xi \) Dimensionless constant that depends on particle shape

\( \rho \) Fluid density (kg/m\(^3\))

\( \rho_f \) Fluid density (kg/m\(^3\))

\( \rho_g \) Gas phase density (kg/m\(^3\))

\( \rho_l \) Liquid phase density (kg/m\(^3\))

\( \rho_p \) Particle density (kg/m\(^3\))

\( \rho_w \) Density of the target material (kg/m\(^3\))

\( \sigma \) Stress tensor (Pa)

\( \sigma_0 \) Target material plastic flow stress (Pa)

\( \sigma_{\text{par}} \) Standard deviations of the particle parallel restitution coefficient

\( \sigma_{\text{per}} \) Standard deviations of the particle perpendicular restitution coefficient

\( \tau_p \) Particle relaxation time (s)

\( \Phi \) Transported variable.
BIBLIOGRAPHY


## APPENDIX

### PREDICTED EROSION IN ELBOWS AND PLUG TEES

**Table A-1. Predicted Erosion (mil/lb) in Elbow with 1-inch Diameter for Water**

<table>
<thead>
<tr>
<th></th>
<th>50 μm</th>
<th>100 μm</th>
<th>150 μm</th>
<th>220 μm</th>
<th>300 μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 ft/s</td>
<td>2.88E-7</td>
<td>3.15E-6</td>
<td>1.34E-5</td>
<td>4.36E-5</td>
<td>2.16E-4</td>
</tr>
<tr>
<td>10 ft/s</td>
<td>1.39E-5</td>
<td>8.49E-5</td>
<td>1.88E-4</td>
<td>6.14E-4</td>
<td>1.89E-3</td>
</tr>
<tr>
<td>20 ft/s</td>
<td>6.76E-4</td>
<td>7.99E-4</td>
<td>1.78E-3</td>
<td>5.03E-3</td>
<td>1.14E-2</td>
</tr>
</tbody>
</table>

**Table A-2. Predicted Erosion (mil/lb) in Elbow with 2-inch Diameter for Water**

<table>
<thead>
<tr>
<th></th>
<th>50 μm</th>
<th>100 μm</th>
<th>150 μm</th>
<th>220 μm</th>
<th>300 μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 ft/s</td>
<td>1.03E-7</td>
<td>1.23E-6</td>
<td>3.42E-6</td>
<td>7.04E-6</td>
<td>1.43E-5</td>
</tr>
<tr>
<td>10 ft/s</td>
<td>3.15E-6</td>
<td>2.05E-5</td>
<td>3.42E-5</td>
<td>6.77E-5</td>
<td>1.29E-4</td>
</tr>
<tr>
<td>20 ft/s</td>
<td>5.75E-5</td>
<td>1.64E-4</td>
<td>2.78E-4</td>
<td>4.82E-4</td>
<td>9.62E-4</td>
</tr>
</tbody>
</table>

**Table A-3. Predicted Erosion (mil/lb) in Elbow with 4-inch Diameter for Water**

<table>
<thead>
<tr>
<th></th>
<th>50 μm</th>
<th>100 μm</th>
<th>150 μm</th>
<th>220 μm</th>
<th>300 μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 ft/s</td>
<td>1.37E-8</td>
<td>2.74E-7</td>
<td>8.08E-7</td>
<td>1.64E-6</td>
<td>2.60E-6</td>
</tr>
<tr>
<td>10 ft/s</td>
<td>9.18E-7</td>
<td>4.11E-6</td>
<td>7.12E-6</td>
<td>1.30E-5</td>
<td>1.83E-5</td>
</tr>
<tr>
<td>20 ft/s</td>
<td>1.16E-5</td>
<td>3.01E-5</td>
<td>4.66E-5</td>
<td>7.16E-5</td>
<td>1.07E-4</td>
</tr>
</tbody>
</table>
Table A-4. Predicted Erosion (mil/lb) in Elbow with 8-inch Diameter for Water

<table>
<thead>
<tr>
<th></th>
<th>50 µm</th>
<th>100 µm</th>
<th>150 µm</th>
<th>220 µm</th>
<th>300 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 ft/s</td>
<td>3.79E-9</td>
<td>4.93E-8</td>
<td>1.64E-7</td>
<td>3.15E-7</td>
<td>4.52E-7</td>
</tr>
<tr>
<td>10 ft/s</td>
<td>1.23E-7</td>
<td>8.22E-7</td>
<td>1.64E-6</td>
<td>2.19E-6</td>
<td>2.62E-6</td>
</tr>
<tr>
<td>20 ft/s</td>
<td>2.05E-6</td>
<td>4.93E-6</td>
<td>6.85E-6</td>
<td>9.86E-6</td>
<td>1.49E-5</td>
</tr>
</tbody>
</table>

Table A-5. Predicted Erosion (mil/lb) in Elbow with 1-inch Diameter for Air

<table>
<thead>
<tr>
<th></th>
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<th>100 µm</th>
<th>150 µm</th>
<th>220 µm</th>
<th>300 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ft/s</td>
<td>9.64E-1</td>
<td>1.14E+0</td>
<td>1.22E+0</td>
<td>1.63E+0</td>
<td>1.32E+0</td>
</tr>
<tr>
<td>100 ft/s</td>
<td>3.42E+0</td>
<td>3.92E+0</td>
<td>4.24E+0</td>
<td>4.34E+0</td>
<td>4.40E+0</td>
</tr>
<tr>
<td>150 ft/s</td>
<td>7.30E+0</td>
<td>8.02E+0</td>
<td>8.52E+0</td>
<td>8.82E+0</td>
<td>8.94E+0</td>
</tr>
</tbody>
</table>

Table A-6. Predicted Erosion (mil/lb) in Elbow with 2-inch Diameter for Air

<table>
<thead>
<tr>
<th></th>
<th>50 µm</th>
<th>100 µm</th>
<th>150 µm</th>
<th>220 µm</th>
<th>300 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ft/s</td>
<td>2.18E-1</td>
<td>2.76E-1</td>
<td>2.90E-1</td>
<td>3.04E-1</td>
<td>3.14E-1</td>
</tr>
<tr>
<td>100 ft/s</td>
<td>7.98E-1</td>
<td>9.36E-1</td>
<td>9.86E-1</td>
<td>1.03E+0</td>
<td>1.06E+0</td>
</tr>
<tr>
<td>150 ft/s</td>
<td>1.71E+0</td>
<td>1.90E+0</td>
<td>1.97E+0</td>
<td>2.12E+0</td>
<td>2.14E+0</td>
</tr>
</tbody>
</table>

Table A-7. Predicted Erosion (mil/lb) in Elbow with 4-inch Diameter for Air

<table>
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<tr>
<th></th>
<th>50 µm</th>
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<th>220 µm</th>
<th>300 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ft/s</td>
<td>3.80E-2</td>
<td>5.80E-2</td>
<td>6.80E-2</td>
<td>7.20E-2</td>
<td>7.40E-2</td>
</tr>
<tr>
<td>100 ft/s</td>
<td>1.56E-1</td>
<td>2.06E-1</td>
<td>2.28E-1</td>
<td>2.38E-1</td>
<td>2.50E-1</td>
</tr>
<tr>
<td>150 ft/s</td>
<td>1.82E-1</td>
<td>2.84E-1</td>
<td>3.58E-1</td>
<td>4.10E-1</td>
<td>4.52E-1</td>
</tr>
</tbody>
</table>
Table A-8. Predicted Erosion (mil/lb) in Elbow with 8-inch Diameter for Air

<table>
<thead>
<tr>
<th></th>
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<th>100 µm</th>
<th>150 µm</th>
<th>220 µm</th>
<th>300 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ft/s</td>
<td>6.82E-3</td>
<td>1.19E-3</td>
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<td>1.77E-2</td>
<td>1.81E-2</td>
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<tr>
<td>100 ft/s</td>
<td>3.02E-2</td>
<td>4.40E-2</td>
<td>5.40E-2</td>
<td>5.64E-2</td>
<td>6.08E-2</td>
</tr>
<tr>
<td>150 ft/s</td>
<td>6.68E-2</td>
<td>9.52E-2</td>
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<td>1.18E-1</td>
<td>1.24E-1</td>
</tr>
</tbody>
</table>

Table A-9. Predicted Erosion (mil/lb) in Plug Tee (End Region) with 1-inch Diameter for Air

<table>
<thead>
<tr>
<th></th>
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<th>100 µm</th>
<th>150 µm</th>
<th>220 µm</th>
<th>300 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ft/s</td>
<td>0.72E-1</td>
<td>0.60E-1</td>
<td>0.54E-1</td>
<td>0.56E-1</td>
<td>0.58E-1</td>
</tr>
<tr>
<td>100 ft/s</td>
<td>2.48E+0</td>
<td>2.06E+0</td>
<td>1.84E+0</td>
<td>1.98E+0</td>
<td>2.06E+0</td>
</tr>
<tr>
<td>150 ft/s</td>
<td>4.74E+0</td>
<td>3.82E+0</td>
<td>3.86E+0</td>
<td>3.98E+0</td>
<td>4.28E+0</td>
</tr>
</tbody>
</table>

Table A-10. Predicted Erosion (mil/lb) in Plug Tee (End Region) with 2-inch Diameter for Air

<table>
<thead>
<tr>
<th></th>
<th>50 µm</th>
<th>100 µm</th>
<th>150 µm</th>
<th>220 µm</th>
<th>300 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ft/s</td>
<td>2.40E-1</td>
<td>2.80E-1</td>
<td>1.78E-1</td>
<td>1.56E-1</td>
<td>1.42E-1</td>
</tr>
<tr>
<td>100 ft/s</td>
<td>1.50E-1</td>
<td>7.20E-1</td>
<td>5.20E-1</td>
<td>5.00E-1</td>
<td>5.00E-1</td>
</tr>
<tr>
<td>150 ft/s</td>
<td>3.78E-1</td>
<td>1.24E+0</td>
<td>1.02E+0</td>
<td>1.00E+0</td>
<td>0.98E-1</td>
</tr>
</tbody>
</table>

Table A-11. Predicted Erosion (mil/lb) in Plug Tee (End Region) with 4-inch Diameter for Air

<table>
<thead>
<tr>
<th></th>
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<th>100 µm</th>
<th>150 µm</th>
<th>220 µm</th>
<th>300 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ft/s</td>
<td>2.20E-2</td>
<td>1.10E-1</td>
<td>8.80E-2</td>
<td>4.80E-2</td>
<td>3.80E-2</td>
</tr>
<tr>
<td>100 ft/s</td>
<td>1.78E-1</td>
<td>4.60E-1</td>
<td>2.20E-1</td>
<td>1.30E-1</td>
<td>1.21E-1</td>
</tr>
<tr>
<td>150 ft/s</td>
<td>6.40E-1</td>
<td>8.40E-1</td>
<td>3.40E-1</td>
<td>2.60E-1</td>
<td>2.40E-1</td>
</tr>
</tbody>
</table>
### Table A-12. Predicted Erosion (mil/lb) in Plug Tee (End Region) with 8-inch Diameter for Air

<table>
<thead>
<tr>
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<th>100 µm</th>
<th>150 µm</th>
<th>220 µm</th>
<th>300 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ft/s</td>
<td>2.74E-3</td>
<td>1.32E-2</td>
<td>2.91E-2</td>
<td>2.52E-2</td>
<td>1.55E-2</td>
</tr>
<tr>
<td>100 ft/s</td>
<td>1.79E-2</td>
<td>8.80E-2</td>
<td>1.18E-1</td>
<td>6.38E-2</td>
<td>4.28E-2</td>
</tr>
<tr>
<td>150 ft/s</td>
<td>4.22E-2</td>
<td>2.46E-1</td>
<td>2.20E-1</td>
<td>1.05E-1</td>
<td>7.84E-2</td>
</tr>
</tbody>
</table>

### Table A-13. Predicted Erosion (mil/lb) in Plug Tee (Side & Corner) with 1-inch Diameter for Water

<table>
<thead>
<tr>
<th></th>
<th>50 µm</th>
<th>100 µm</th>
<th>150 µm</th>
<th>220 µm</th>
<th>300 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 ft/s</td>
<td>1.06E-6</td>
<td>1.60E-5</td>
<td>6.08E-5</td>
<td>2.28E-4</td>
<td>6.02E-4</td>
</tr>
<tr>
<td>10 ft/s</td>
<td>3.42E-5</td>
<td>2.11E-4</td>
<td>6.10E-4</td>
<td>1.91E-3</td>
<td>3.90E-3</td>
</tr>
<tr>
<td>20 ft/s</td>
<td>7.53E-4</td>
<td>1.73E-3</td>
<td>4.38E-3</td>
<td>1.03E-2</td>
<td>1.84E-2</td>
</tr>
</tbody>
</table>

### Table A-14. Predicted Erosion (mil/lb) in Plug Tee (Side & Region) with 2-inch Diameter for Water

<table>
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<tr>
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<th>100 µm</th>
<th>150 µm</th>
<th>220 µm</th>
<th>300 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 ft/s</td>
<td>3.63E-7</td>
<td>2.74E-6</td>
<td>6.85E-6</td>
<td>1.88E-5</td>
<td>4.60E-5</td>
</tr>
<tr>
<td>10 ft/s</td>
<td>1.16E-5</td>
<td>4.52E-5</td>
<td>7.26E-5</td>
<td>1.51E-4</td>
<td>3.16E-4</td>
</tr>
<tr>
<td>20 ft/s</td>
<td>1.44E-4</td>
<td>2.66E-4</td>
<td>4.47E-4</td>
<td>8.22E-4</td>
<td>1.71E-3</td>
</tr>
</tbody>
</table>
Table A-15. Predicted Erosion (mil/lb) in Plug Tee (Side & Corner Region) with 4-inch Diameter for Water

<table>
<thead>
<tr>
<th></th>
<th>50 µm</th>
<th>100 µm</th>
<th>150 µm</th>
<th>220 µm</th>
<th>300 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 ft/s</td>
<td>6.85E-8</td>
<td>6.85E-7</td>
<td>1.78E-6</td>
<td>3.77E-6</td>
<td>5.48E-6</td>
</tr>
<tr>
<td>10 ft/s</td>
<td>3.15E-6</td>
<td>1.32E-5</td>
<td>2.26E-5</td>
<td>3.85E-5</td>
<td>5.83E-5</td>
</tr>
<tr>
<td>20 ft/s</td>
<td>4.27E-5</td>
<td>8.82E-5</td>
<td>8.39E-5</td>
<td>1.22E-4</td>
<td>1.98E-4</td>
</tr>
</tbody>
</table>

Table A-16. Predicted Erosion (mil/lb) in Plug Tee (Side & Corner Region) with 8-inch Diameter for Water

<table>
<thead>
<tr>
<th></th>
<th>50 µm</th>
<th>100 µm</th>
<th>150 µm</th>
<th>220 µm</th>
<th>300 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 ft/s</td>
<td>2.05E-8</td>
<td>2.60E-7</td>
<td>6.58E-7</td>
<td>1.22E-6</td>
<td>1.64E-6</td>
</tr>
<tr>
<td>10 ft/s</td>
<td>6.51E-7</td>
<td>3.63E-6</td>
<td>5.41E-6</td>
<td>6.85E-6</td>
<td>7.53E-6</td>
</tr>
<tr>
<td>20 ft/s</td>
<td>9.59E-6</td>
<td>1.92E-5</td>
<td>2.47E-5</td>
<td>2.79E-5</td>
<td>3.58E-5</td>
</tr>
</tbody>
</table>

Table A-17. Predicted Erosion (mil/lb) in Plug Tee (Side & Corner Region) with 1-inch Diameter for Air

<table>
<thead>
<tr>
<th></th>
<th>50 µm</th>
<th>100 µm</th>
<th>150 µm</th>
<th>220 µm</th>
<th>300 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ft/s</td>
<td>3.02E-1</td>
<td>2.56E-1</td>
<td>1.32E-1</td>
<td>7.20E-2</td>
<td>5.62E-2</td>
</tr>
<tr>
<td>100 ft/s</td>
<td>1.27E+0</td>
<td>3.76E-1</td>
<td>2.94E-1</td>
<td>1.99E-1</td>
<td>2.06E-1</td>
</tr>
<tr>
<td>150 ft/s</td>
<td>3.16E+0</td>
<td>1.03E+0</td>
<td>5.48E-1</td>
<td>4.24E-1</td>
<td>4.01E-1</td>
</tr>
</tbody>
</table>
Table A-18. Predicted Erosion (mil/lb) in Plug Tee (Side & Corner Region) with 2-inch Diameter for Air

<table>
<thead>
<tr>
<th></th>
<th>50 µm</th>
<th>100 µm</th>
<th>150 µm</th>
<th>220 µm</th>
<th>300 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ft/s</td>
<td>1.23E-1</td>
<td>2.24E-4</td>
<td>1.09E-1</td>
<td>4.80E-1</td>
<td>2.54E-2</td>
</tr>
<tr>
<td>100 ft/s</td>
<td>9.95E-1</td>
<td>5.90E-1</td>
<td>2.26E-1</td>
<td>1.04E-1</td>
<td>5.48E-2</td>
</tr>
<tr>
<td>150 ft/s</td>
<td>2.94E+0</td>
<td>9.08E-1</td>
<td>3.50E-1</td>
<td>1.73E-1</td>
<td>1.06E-1</td>
</tr>
</tbody>
</table>

Table A-19. Predicted Erosion (mil/lb) in Plug Tee (Side & Corner Region) with 2-inch Diameter for Air

<table>
<thead>
<tr>
<th></th>
<th>50 µm</th>
<th>100 µm</th>
<th>150 µm</th>
<th>220 µm</th>
<th>300 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ft/s</td>
<td>1.96E-2</td>
<td>7.54E-2</td>
<td>7.46E-2</td>
<td>3.84E-2</td>
<td>1.71E-2</td>
</tr>
<tr>
<td>100 ft/s</td>
<td>1.21E-1</td>
<td>3.92E-1</td>
<td>2.08E-1</td>
<td>7.54E-2</td>
<td>3.50E-2</td>
</tr>
<tr>
<td>150 ft/s</td>
<td>4.80E-1</td>
<td>7.88E-1</td>
<td>3.16E-1</td>
<td>1.25E-1</td>
<td>6.36E-2</td>
</tr>
</tbody>
</table>

Table A-20. Predicted Erosion (mil/lb) in Plug Tee (Side & Corner Region) with 8-inch Diameter for Air

<table>
<thead>
<tr>
<th></th>
<th>50 µm</th>
<th>100 µm</th>
<th>150 µm</th>
<th>220 µm</th>
<th>300 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ft/s</td>
<td>6.44E-3</td>
<td>4.26E-3</td>
<td>1.68E-2</td>
<td>2.02E-2</td>
<td>1.37E-2</td>
</tr>
<tr>
<td>100 ft/s</td>
<td>1.42E-2</td>
<td>5.34E-2</td>
<td>8.90E-2</td>
<td>6.38E-2</td>
<td>3.28E-2</td>
</tr>
<tr>
<td>150 ft/s</td>
<td>3.42E-2</td>
<td>1.61E-1</td>
<td>1.85E-1</td>
<td>1.05E-1</td>
<td>5.48E-1</td>
</tr>
</tbody>
</table>